5.4.2007

Cost and Pricing in a Regime of Sustainable Water Resources

Or Goldfarb and Yoav Kislev

Abstract

The article offers a mathematical and numerical programming analysis of cost and pricing of freshwater and effluent in a sustainable regime. Since demand in the urban sector is inelastic, the program focuses on agriculture. Sustainability is maintained by limiting withdrawal from natural sources to “safe yield” and by removing all salts added annually to the aquifers. Consistently, the mathematical model is formulated in terms of a steady state. The model yields marginal cost of fresh and recycled water and corresponding prices that incorporate both conventional cost items and the cost of removing salts.

The withdrawal of water and its conveyance, sewage disposal, irrigation with freshwater and effluent—all interfere in the natural water cycle and influence the quantity and quality of the resources. A sustainable regime will maintain acceptable quantity and quality of water. This may entail constraining of withdrawals to safe yields (water is not mined) and costly artificial removal of contaminations. In the article we identify the implied cost of water from its sources and the corresponding pricing of fresh and recycled water.

Since the demand for water to be used in the urban sector is inelastic, the sector is a price taker. Consequently, the major questions of allocation and pricing pertain to agriculture, and they are discussed in the paper in two parts. The first is a mathematical analysis of cost and pricing; the second part is an illustrative application to a regional water economy in Israel. The mathematical model is also presented in the terms of the illustration. The analysis is preceded by a short review of the water economy in Israel and the region (for a detailed survey, see Kislev, 2006).

The water economy

Israel is a small and narrow country; half of its area is desert. Precipitation, only in the winter, averages more than 700 mm per year in the north and less than 35 mm in the southern tip of the country. The core functions of the water sector have been to store water from winter to summer and from rainy to dry years and to carry water from the north to the center and the south. With expanding population and growing urbanization, sewage treatment and recycled water are growing in importance and seawater desalination is being introduced.

Fresh water is stored in Lake Kinneret (Sea of Galilee) and in several groundwater reservoirs; the largest two are the Mountain and the Coastal aquifers, both stretching from north to south in parallel to the coast of the Mediterranean Sea. The discussion in the paper is presented in terms of the Coastal region, the area above the Coastal aquifer. The country’s largest urban centers, including Tel Aviv, are located along the coast and they draw water locally from the region’s aquifer.

Additional quantities come from the Kinneret and the Mountain Aquifer; and in the future, from desalination plants. Figure 1 depicts cost of freshwater in the Coast as seen in the analysis; the supply is represented by a step-function with constant per unit cost by source. [The units of water in the article are cubic meter, MC, and million cubic meters, MCM. The monetary unit is New Israeli Sheqel at the exchange rate of NIS 4.5 = 1 US dollar.]

As the demanded quantities of water grow, extraction moves to relatively costly sources and the scarcity value—attributable to the sources whose capacity was exhausted—increases. An extraction levy is imposed on suppliers in Israel in an attempt to reflect the scarcity value of the sources they withdraw from.

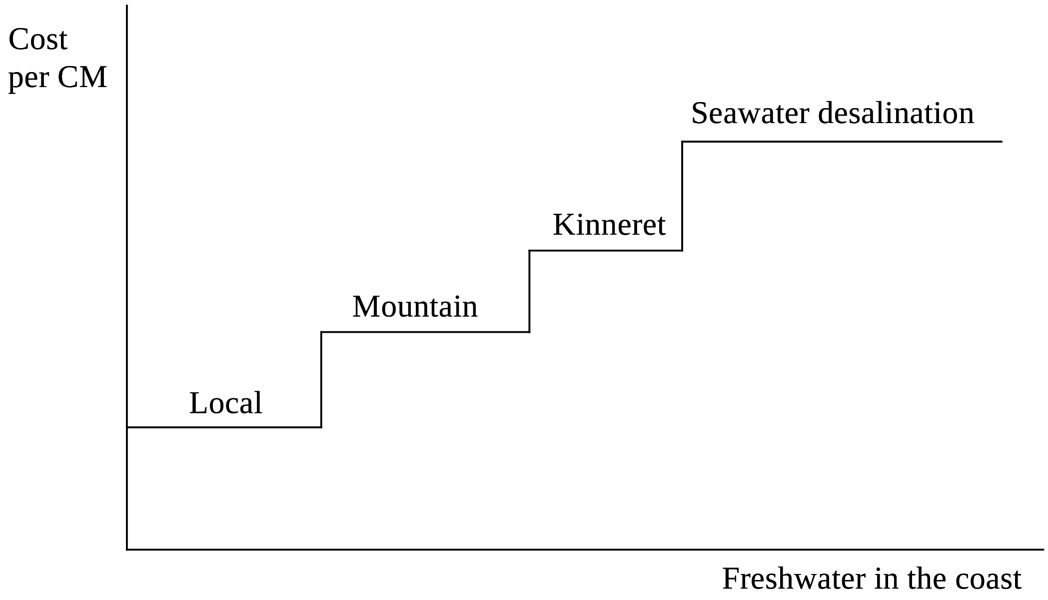

Figure 1. Cost function for freshwater in the Coastal area

Agriculture in the Coastal region is also irrigated with water from the country’s reservoirs and with recycled effluent. Water and effluent carry salts that gradually sink into the Coastal aquifer and accumulate in its water. In addition, salt is reaching the reservoir from sources we term autonomous (unrelated to the activities of the water economy): underground flows and spray form the Mediterranean Sea. In a sustainable regime salts will eventually have to be prevented from reaching the aquifer or removed from its water by desalination. This eventuality is incorporated in the analysis of the article. (See Hillel, 2000, for a survey of salinity problems in irrigation and a review of their economic analysis.)

A mathematical discussion

This section presents a mathematical programming model of the Coastal region, part of the complete model encompassing the whole country. The model incorporates freshwater provision from the Coastal aquifer, the Mountain aquifer, and seawater desalination (for brevity, we disregard Lake Kinneret) and also the use of effluent in agriculture. There are two consuming sectors in the model—urban and agriculture. The urban sector receives a predetermined quantity of water and a given ratio of the water used in the sector is collected as sewage and treated. The recycled effluent can be used in agriculture or released to the sea. As indicated, irrigation deposits salts on the ground and eventually the salts reach the water in the aquifer. Desalination is used to prevent salt content from rising above a given concentration.

The programming is seen as applying to a future date and the sustainability requirement is maintained in formulating the analysis for a steady state: constant quantities of salts are added annually to the aquifer and identical quantities are removed—drained to the sea or removed by desalination of water from the reservoir. The country's economy is not expected to be in a steady state—population and income are growing—but future changes in demand for water are to be satisfied with expanding supply of desalinated seawater. It is therefore appropriate to view the sub-sector of the natural resources as independent and adopt a steady state model to express formally the sustainability requirement.

Agricultural production is in the model a function of the amounts of water and effluent, the contribution of the latter being only a fraction of the productive contribution of freshwater. The objective function of the program is a measure of net income of the water sector: value of production in agriculture *minus* total cost of the supply of water and effluent and the cost of sewage removal. This definition disregards important benefit and cost elements but, since most neglected elements do not change between programming alternatives, their omission does not affect the choice of the plan and its implications and the maximization of the objective function is analogous to maximizing the contribution of the water sector to the national income.

Functions and variables

F( )	A well-behaved production function in agriculture (NIS per year)
f( )	Value of marginal product in agriculture (NIS per CM)
B	Constraint or requirement of provision (CM per year)
M	Freshwater (CM per year)
R	Effluent (CM per year)
μ	Salt concentration in water (gram chlorine per CM)
δ	Addition of salts to effluent (gram per CM per year)
Δ	Autonomous addition of salts to the aquifer (grams per year)
λ, ϕ	Lagrange multipliers (shadow prices)
σ	Value of effluent in agriculture relative to freshwater
ρ	Ratio of effluent in urban water
C	Cost (NIS per CM)
P	Price (NIS per CM)
E	Extraction levy (NIS per CM).

Indexes

a	Agriculture
u	Urban

y	Mountain
h	Coastal region or aquifer
dh	Desalination of Coastal water
dy	Desalination of Mountain water
t	Desalination of seawater
m	Water
r	Effluent
s	Sea (for removed effluent).

Salt

Salts are added to the Coastal aquifer from several sources. The autonomous quantity Δ_h grams is added yearly; μ_y is the concentration of salt in the Mountain water and it is therefore also the amount added per CM of the water from this source used in the Coastal region; δ_r is the amount of salt *added* in urban use per CM of effluent. Desalination is not complete: desalinated water carries 20 grams of chloride per CM and this quantity is also added to the reservoir when this water is used.

Salts deposited on the ground move to the aquifer with a delay, in some cases of many years. The delay is ignored here—in a steady state the amount of salts deposited this year is equal to the amount deposited in the distant past and reaching the groundwater today. A sustainable policy dictates continuous removal of the salts at a pace identical to their deposit.

Salts may be removed from the Coastal region or prevented from reaching it, either by desalination of the aquifer's water and the water imported to the region from the Mountain aquifer or removal of effluent to the sea. In the last case, the amount removed per CM is the salt content of water supplied to urban users, μ_u, *plus* the salt added, δ_r. The concentration μ_u is considered here as a given constant; in the application to follow it is calculated as a weighted average of the salt content of water supplied from various sources.

Resources, uses, and constraints

The requirements of the program are expressed as three sets of constraints.

Equality constraints

Freshwater: provision to agriculture and the urban sector is equal to supply from the Coastal aquifer, Mountain, and seawater desalination: $M_a + M_u = M_h + M_y + M_t$.

Effluent: supply to agriculture plus removal to the sea is equal to the quantity collected and treated in the urban sector: $R_a + R_s = \rho M_u$.

Salt: the quantity added to the Coastal region is eliminated by desalination of the aquifers' water or removal of the effluent to the sea:

$$\Delta_h + \mu_y M_y + \delta_r \rho M_u + 20M_t = (\mu_h - 20)M_{dh} + (\mu_y - 20)M_{dy} + (\mu_u + \delta_r)R_s .$$

(Coastal water does not appear on the left hand side of the constraint; this source does not *add* salts to the region.)

Urban water supply requirement: $M_u = B_u$.

Inequalities

Coastal Aquifer, extraction	$M_h \le B_h$
Mountain, extraction	$M_y \le B_y$
Coast, desalination	$M_{dh} \le M_h$
Mountain, desalination	$M_{dy} \le M_y$

The first two inequalities limit withdrawal to safe yield; the others limit desalination to the water withdrawn from the aquifers.

Nonnegativity

All the quantity variables are nonnegative.

The Programming Problem and the First Order Conditions

Equation (1) is the Kuhn-Tucker Lagrangian of the programming problem. Following Simon and Blume (1994), we write specific multipliers for the constraints: λ for equalities and ϕ for inequalities. In this formulation, the quantities are the primal variables, they are the activities of the program, and the Lagrange multipliers are the dual variables.

$$
\begin{aligned}
L = {} & F(M_a + \sigma R_a) - C_h M_h - C_y M_y - C_t M_t - C_a R_a - C_s R_s \\
& - C_d (M_{dh} + M_{dy}) - \lambda_m (M_a + M_u - M_h - M_y - M_t) \\
& - \lambda_r (R_a + R_s - \rho M_u) - \lambda_d [\Delta_h + \mu_y M_y + 20 M_t + \delta_r \rho M_u \\
& - (\mu_h - 20) M_{dh} - (\mu_y - 20) M_{dy} - (\mu_u + \delta_r) R_s] - \lambda_u (B_u - M_u) \\
& - \phi_h (M_h - B_h) - \phi_y (M_y - B_y) - \phi_{dh} (M_{dh} - M_h) - \phi_{dy} (M_{dy} - M_y)
\end{aligned}
\tag{1}
$$

First order conditions

Equalities

(2)

$$a.\quad \frac{\partial L}{\partial \lambda_m} = M_a + M_u - M_h - M_y - M_t = 0$$

$$b.\quad \frac{\partial L}{\partial \lambda_r} = R_a + R_s - \rho M_u = 0$$

$$c.\quad \frac{\partial L}{\partial \lambda_d} = \Delta_h + \mu_y M_y + 20M_t + \delta_r \rho M_u - (\mu_h - 20)M_{dh} - (\mu_y - 20)M_{dy} - \left(\mu_u + \delta_r\right) R_s = 0$$

$$d.\quad \frac{\partial L}{\partial \lambda_u} = B_u - M_u = 0$$

Inequalities

(3)

$$a.\quad \phi_h \frac{\partial L}{\partial \phi_h} = \phi_h \left[M_h - B_h \right] = 0$$

$$b.\quad \phi_y \frac{\partial L}{\partial \phi_y} = \phi_y \left[M_y - B_y \right] = 0$$

$$c.\quad \phi_{dh} \frac{\partial L}{\partial \phi_{dh}} = \phi_{dh} \left[M_{dh} - M_h \right] = 0$$

$$d.\quad \phi_{dy} \frac{\partial L}{\partial \phi_{dy}} = \phi_{dy} \left[M_{dy} - M_y \right] = 0$$

Nonnegativities

$$a.\quad M_a \frac{\partial L}{\partial M_a} = M_a \left[f(M_a + \sigma R_a) - \lambda_m \right] = 0$$

$$b.\quad R_a \frac{\partial L}{\partial R_a} = R_a \left[\sigma f(M_a + \sigma R_a) - C_a - \lambda_r \right] = 0$$

$$c.\quad M_h \frac{\partial L}{\partial M_h} = M_h \left[-C_h + \lambda_m - \phi_h + \phi_{dh} \right] = 0$$

$$d.\quad M_y \frac{\partial L}{\partial M_y} = M_y \left[-C_y + \lambda_m - \lambda_d \mu_y - \phi_y + \phi_{dy} \right] = 0$$

$$e.\quad M_t \frac{\partial L}{\partial M_t} = M_t \left[-C_t + \lambda_m - 20\lambda_d \right] = 0$$

$$f.\quad R_s \frac{\partial L}{\partial R_s} = R_s \left[-C_s - \lambda_r + \lambda_d \left(\mu_u + \delta_r \right) \right] = 0$$

$$g.\quad M_{dh} \frac{\partial L}{\partial M_{dh}} = M_{dh} \left[-C_d + \lambda_d \left(\mu_h - 20 \right) - \phi_{dh} \right] = 0$$

$$h.\quad M_{dy} \frac{\partial L}{\partial M_{dy}} = M_{dy} \left[-C_d + \lambda_d \left(\mu_y - 20 \right) - \phi_{dy} \right] = 0$$

(4)

$$i.\quad M_u \frac{\partial L}{\partial M_u} = M_u \left[-\lambda_m + \lambda_r \rho - \lambda_d \delta_r \rho + \lambda_u \right] = 0$$

The expressions in the brackets in (2)-(4) are zero or negative.

Alternative Solutions

A solution of the programming problem is a set optimal allocations and the associated multipliers. The problem stated in (1) may have a host of alternative solutions depending on cost and on the profitability of agriculture. The major mover of the solutions is the quantity of water demanded in the Coastal region. If this quantity is small, it will be supplied just from local withdrawals and no use will be made of other sources; if the quantity demanded is larger, the program will turn to the relatively expensive sources: the Mountain aquifer or even seawater desalination. Likewise effluent may be utilized in the region’s agriculture or disposed into the sea. We examine three cases.

Case I

In this case, the solution of the programming problem calls for extraction of water from the Coastal aquifer up to its constraint, B_h, and for additional quantities to come from the Mountain aquifer. Seawater is not desalinated. Part of the water withdrawn from the Coastal aquifer is desalinated; the desalination constraint in the Coast and the extraction constraint in the Mountain aquifer are not met. Freshwater is supplied to the urban sector and to agriculture, effluent is supplied to agriculture; effluent is not removed to the sea.

Writing formally, with the numbers of the corresponding first order conditions in parentheses,

(5)
$$
\begin{aligned}
&M_a > 0 \quad (4.a)\\
&R_a = \rho M_u > 0 \quad (4.b)\\
&M_h = B_h > 0 \quad (3.a)\\
&0 < M_y < B_y \quad (4.d),(3.b)\\
&0 < M_{dh} < M_h \quad (4.g),(3.c)\\
&M_u = B_u > 0 \quad (4.i)
\end{aligned}
$$

The value of the primal variables not appearing in (5) is zero.

Rearranging terms in (4), the following multipliers are factored out

(6)
$$
\begin{aligned}
\lambda_d &= C_d /(\mu_h - 20)\\
\lambda_m &= f(M_a + \sigma R_a) = C_y + C_d \mu_y /(\mu_h - 20)\\
\phi_h &= f(M_a + \sigma R_a) - C_h\\
\lambda_r &= \sigma f(M_a + \sigma R_a) - C_a\\
\lambda_u &= \lambda_m - \lambda_r \rho + \lambda_d \delta_r \rho\\
&= f(M_a + \sigma R_a)(1 - \rho\sigma) + \rho[C_a + C_d \delta_r /(\mu_h - 20)]
\end{aligned}
$$

The first shadow price in (6) is of desalination Coastal water. It is the cost of desalination of one CM divided by the amount of salt removed; that is, λ_d is the cost of salt removal per gram of chloride.

The multiplier in the second equation in (6), λ_m, is the Value of the Marginal Product of water in the Coastal area agriculture and it is also equal to the marginal cost of water provision. The cost of moving water from the Mountain aquifer to the Coastal region is larger than the cost of local extraction. Hence, if in the solution of (1) water is moved, the cost of freshwater, λ_m, is the cost of the Mountain's water at the Coast and this magnitude is equal to the cost of moving the water from the remote aquifer *plus* the cost of removing the salts imported with this water.

The third equation defines the scarcity value of Coastal water, ϕ_h. It is equal to the VMP of water *minus* cost of extraction; in other words, to the cost of water from the Mountain *minus* extraction at the Coast.

The multiplier λ_r is the scarcity value of the effluent; it is its VMP (water's multiplied by σ) *minus* the cost of recycling. This scarcity value is the net return the urban sector receives for providing agriculture with the effluent. The cost of water to the urban sector is the opportunity cost of freshwater in agriculture *minus* the value of the effluent the town transfers to the farm sector (per CM of water); that is, the program visualizes the urban sector as purchasing water, treating the sewage, removing salts, and selling the effluent to farmers at a price equal to its VMP. The maxim followed is not *the polluter pays* but rather, *the polluter is responsible*. (Feinerman et al, 2001, reached a similar conclusion and the point is illuminated further below.)

Given the above multipliers and adopting the principle of marginal cost pricing, the prices and the extraction levy (the scarcity cost) will be

$$
(7)\qquad
\begin{aligned}
&P_a = \lambda_m = f(M_a + \sigma R_a) = C_y + C_d \mu_y / (\mu_h - 20) \\
&P_r = \sigma P_a \\
&P_u = \lambda_u \\
&E_h = \phi_h
\end{aligned}
$$

By the first line in (7) farmers (and urban users) pay for the transfer of water from the Mountain and also for the removal of the salts carried by the water from this source. The last attribute will be modified in the next case.

Cost recovery

The cost function being linear, payment of marginal cost prices for water and effluent covers all cost *except* the cost of the removal of the autonomous quantity of salt, Δ_h :

$$
(8)\qquad
\begin{aligned}
&P_a M_a + P_u M_u + P_r R_a \\
&= \left(M_a + M_u\right)\left[C_y + C_d \mu_y / \left(\mu_h - 20\right)\right] \quad + R_a \left[C_a + C_d \delta_r / \left(\mu_h - 20\right)\right] \\
&= \left(C_h + E_h\right) M_h + C_y M_y + C_a R_a + C_d M_{dh} - C_d \Delta_h / \left(\mu_h - 20\right)
\end{aligned}
$$

The first line in (8) is the payment of water and effluent by users. The last line is cost by item: Coastal water (including the extraction levy), Mountain water, effluent, and salt removal, *except* the cost of removing the autonomous salt. [The ratio $\Delta_h / \left(\mu_h - 20\right)$ is the number of CMs that have to be desalinated to remove the autonomous quantity of salt.] Two equalities, copied from the corresponding first order conditions, were utilized in moving from the first to the last line in (8),

$$
(9)\qquad
\begin{aligned}
&M_a + M_u = M_h + M_y \quad (2.a) \\
&M_{dh} = \left(\Delta_h + \mu_y M_y + \delta_r R_a\right) / \left(\mu_h - 20\right) \quad (2.c)
\end{aligned}
$$

The separate component, $C_d \Delta_h / \left(\mu_h - 20\right)$ in (8), is part of the cost of the water economy but it is not covered by users' pay. In principle, the extraction levy, $E_h M_h$, is a tax to be paid to the country's treasury, not to be kept in the water sector. But there is a connection: the autonomous salts can be seen a public "bad" and their removal a public service. The associated cost, and hence the utilization of the Coastal aquifer, is warranted only if it is smaller than the scarcity value.

Case II

This case corresponds to higher profitability in agriculture than in Case I and to a solution calling for larger quantities of water: the extraction constraint in the Mountain aquifer is met, and seawater is desalinated. Formally, we add to (5)

$$(10)\quad \begin{array}{l} M_y = B_y > 0 \quad (4.d), (3.b) \\ M_t > 0 \quad (4.e) \end{array}$$

The shadow price of freshwater is now

$$(11)\quad \lambda_m = f(M_a + \sigma R_a) = C_t + 20C_d / (\mu_h - 20)$$

The marginal cost of desalinated water is the cost of desalination *plus* the cost of removing the (small amounts of) salts the desalinated water adds to the aquifer.

The other multipliers will be in this case

$$(12)\quad \begin{array}{l} \lambda_d = C_d /(\mu_h - 20) \\ \phi_h = C_t + 20C_d / (\mu_h - 20) - C_h \\ \phi_y = C_t + 20C_d / (\mu_h - 20) - C_y - C_d \mu_y /(\mu_h - 20) \\ \lambda_r = \sigma\left[C_t + 20C_d / (\mu_h - 20)\right] - C_a \\ \lambda_u = \left[C_t + 20C_d / (\mu_h - 20)\right](1 - \rho\sigma) + \rho[C_a + C_d \delta_r /(\mu_h - 20)] \end{array}$$

Again, the prices are $P_a = \lambda_m$, $P_r = \sigma P_a$, $P_u = \lambda_u$, $E_h = \phi_h$, $E_y = \phi_y$. Here, in Case II, the scarcity value of the Mountain water, ϕ_y, is positive; it was zero in Case I. Unlike in the previous case—where the cost of Mountain water at the Coast was higher the higher its salt content—now the cost of removing salts carried from the Mountain is shouldered by the public at large (the fisc). To see this, examine the components of ϕ_y, the higher the concentration of salts in the Mountain's water (μ_y) the lower the scarcity value. Users pay one price for freshwater from any source, λ_m, whether salt concentration in the Mountain water is zero or a positive magnitude. Whereas the government collects a lower extraction levy the higher the concentration of salts in the Mountain water. In simple terms, the government provides the Coastal users with water of low quality (salty) and is paid accordingly.

Case III

This case is presented to examine the possibility of discharging part of the effluent into the sea in addition to delivering effluent to agriculture. Effluent will be removed to the sea if, in the solution, the supply of effluent is large relative to the demand in agriculture. To (5) and (10) we add: $R_S \geq 0$. With the possibility of discharging effluent into the sea, the doublet (pair) (R_a,R_s) can take the values (+,0), (0,+), (+,+), a + sign marking nonzero value. The doublet (0,0) is not possible—the effluent must be discharged somewhere. With effluent discharge both to agriculture and into the sea (+,+) its marginal net benefit, λ_r, is defined both in (4.*b*) and in (4.*f*). To distinguish, we write

$$\lambda_{ra} = \sigma f(M_a + \sigma R_a) - C_a$$
(13)
$$\lambda_{rs} = -C_s + C_d\left(\mu_u + \delta_r\right) / \left(\mu_h - 20\right)$$

Notice that λ_{ra} is a function of the amount of water and effluent while, given the parameters of the problem, λ_{rs} is constant. Consequently, when effluent is both used in agriculture and discharged into the sea

(14) $R_a > 0, R_s > 0 \quad \lambda_{ra} = \lambda_{rs}$

Figure 2 illustrates. In the figure, R_0 is the total quantity of effluent; $\sigma f\left(M_a + \sigma R_a\right)$ is *gross* VMP of the effluent in agriculture; the curve λ_{ra} is the net contribution; net of sewage treatment cost. For $\lambda_{rs} = \lambda_{ra}$, both R_a and R_s are positive. The quantity R_a is delivered to agriculture and R_s=R_0-R_a is removed to the sea.

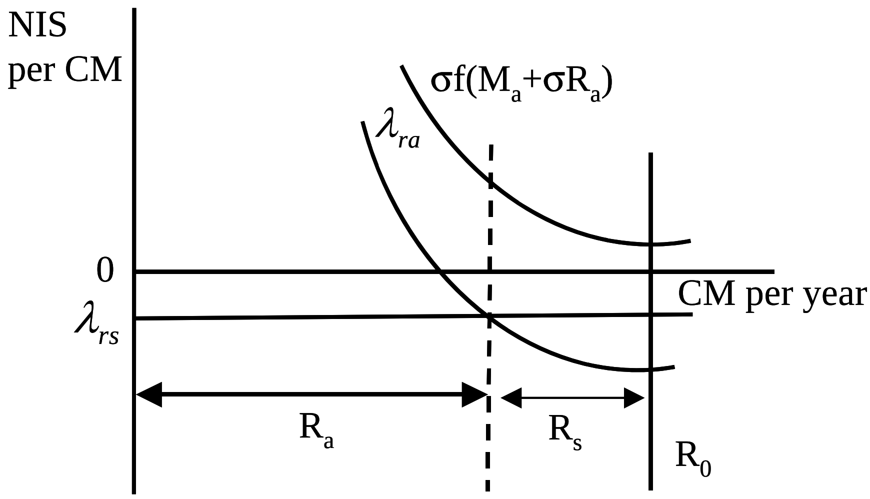

Figure 2. Allocation of Effluent to Agriculture and Sea

The removal of effluent to the sea does not contribute to agricultural production, but it does contribute economically by deleting salts from the Coastal area. In the solution depicted in the diagram, the net contribution of the effluent in agriculture is negative, but so long as this negative (marginal) contribution is not smaller (larger in absolute values) than the net cost of removal to the sea, effluent is delivered to agriculture. (The marginal contributions are positive in the numerical application below.)

The price of the effluent is $P_r = \sigma f\left(M_a + \sigma R_a\right)$ and it may be "subsidized" in the sense that the farmers do not pay the full cost of sewage treatment and salt removal; urban water users cover this cost, see λ_u in (6). The VMP of effluent in agriculture may even be negative; farmers will then be paid to take it. But, with a well-behaved production function, always VMP>0, as in Figure 2.

Application

The application of the model presented above is set for the year 2020 and covers 17 of the agricultural regions in Israel. The 17 regions are interconnected by a system of water supply and effluent removal. Four other regions, not connected to the national grid, are excluded from the programs. The objective function maximized is the net return of the water economy defined as the value of production in agriculture *minus* total cost in the water and sewage sector. Agricultural production is a function of water use, other factors are taken as constants; and, as this function is not linear, the programming is also nonlinear.

Formally, the programming model is: choose x_i (i=1,2, …, 109) to maximize z in

(15) $$z = \sum_{i=1}^{17} F_i\left(x_i\right) - \sum_{i=19}^{109} c_i x_i$$

subject to

(16) $$\begin{aligned} &\mathbf{Ax} \leq \mathbf{B} \\ &\mathbf{x} \geq \mathbf{0} \end{aligned}$$

where **A** is a 170×109 matrix of coefficients and $\mathbf{x}$ is a 109 vector of activities; **B** is a 170 vector of constrains.

As in the mathematical discussion above, in equation (15) $F_i\left(\;\right)$ is the region's *i* production function and x_i (i=1, ... , 17) is the quantity of water, $M_i + \sigma R_i$, with $\sigma = 0.80$ (by assumption relying on judgment of extension specialists). Effluent is defined in the application to cover both recycled sewage and other types of marginal (non-potable) water. The calibration of the production function is presented the Appendix.

The 109 activities, quantity variables in the model, include extraction, supply to agriculture and to the urban sector, transferring water between regions, desalination of brackish and Coastal water, desalination of seawater, effluent removal and provision to agriculture, and others. There are two kinds of constraints in the model: 61 maximum and equality constraints; among them, the definition of the first 17 x_i variables (water *plus* effluent), the given quantities provided to the urban sector in the regions, the quantity of effluent collected in town is equal to the quantity provided to agriculture *plus* the quantity removed to the sea, the extraction from a source does not exceed the safe yield, the quantity of water desalinated from a source (such as desalination to remove salts from the coastal aquifer) does not exceed the quantity extracted from the same source. We assume in the construction of the program that by the year 2020 the Coastal aquifer will have reached its steady state salt concentration of 250 grams chloride per CM (the concentration in the Mountain aquifer is expected to stay at the present level of 160 grams per CM).

Cost

The costs were region specific similarly to the function presented in Figure 1: supply of freshwater and effluent from local source, conveyance of water (fresh and effluent) from other regions, seawater desalination NIS 2.7 per CM, desalination of brackish and freshwater NIS 0.90 per CM.

Programming Alternatives

The program was conducted for several different alternatives. The two major alternatives were:
a. *Economic*, maximizing the contribution of the water sector to national income;
b. *Agriculture*, regional water allocations (freshwater and effluent) were set as specified by the plans of the Ministry of Agriculture and water prices were set to clear the markets. They were thus subsidized.

Supply and allocation of water and effluent are presented, by alternative, in Table 1. In the agricultural alternative, the sector is allocated more water than in the economic alternative. Consequently, larger quantities of seawater are desalinated and effluent is not discharged into the sea. Water losses occur in conveyance between regions. There is less conveyance in the agricultural alternative and relatively smaller losses. In the following we exemplify allocation and marginal cost for one region.

Table 1. Water Supply and Uses in Two Programming Alternatives in 2020, million CM

Alternative	Economic	Agriculture
Supply		
Fresh natural	1355	1371
Desalinated seawater	127	485
Effluent	686	686
Total	2,168	2,542
Uses		
Urban, freshwater	1,344	1,344
Agriculture		
Freshwater	78	474
Effluent	598	686
Total	646	1,160
Into the sea, effluent	88	-
Water losses	60	38
Total	2,168	2,542

Note: Coastal water desalination, to remove salts, is 2 MCM in the economic alternative and 52 MCM in the agricultural alternative. These quantities are included in the supply of freshwater.

Table 2. Supply and Uses in Region α in 2020, Economic Alternative, million CM.

Supply		Uses	
Coastal aquifer	21	*Urban*, freshwater	65
Mountain aquifer	48	*Agriculture*	
Effluent	77.9	Freshwater	4
		Effluent	36.8
		Export, effluent	
		To the sea	6.8
		To region β	29.6
		To region γ	4.7
Total	146.9		146.9

Notes:
a. The regions are: α Hadera, β Ra'anana, γ Rehovot.
b. Salt content in the Mountain water and the (steady state) Coastal aquifer is, respectively, 160 and 250 grams chloride per CM.

The numerical program differs from the algebraic model presented above in two ways: it is multi-regional with water and effluent transferred between regions; and salt concentration in urban water is, in the program, a weighted average of the concentration of salts in the water of the sources provided to towns. We exemplify with a single region in the Coastal area, region α in Table 2. The region receives water from the Coastal and the Mountain aquifers (21 and 48 MCM, respectively) and transfers effluent to regions β and γ (29.6 and 4.7 MCM) in addition to disposing 6.8 MCM of effluent into the sea. To see the effect of the incorporation of several regions and average salt concentration in the program, we reformulate eq. (1) and write it as

$$
\begin{aligned}
L_{\alpha} = {} & F(M_a + \sigma R_a) - C_h M_h - C_y M_y - C_a R_a - C_s R_s - C_d M_{dh} \\
& - \lambda_m (M_a + M_u - M_h - M_y) - \lambda_r (R_a + R_s + R_{\alpha\beta} + R_{\alpha\gamma} - \rho M_u) \\
& - \lambda_d [\mu_y M_y + \delta_r \rho M_u - (\mu_h - 20) M_{dh} - (\mu_u + \delta_r)\left(R_s + R_{\alpha\beta} + R_{\alpha\gamma}\right)] \\
& - \lambda_u (B_u - M_u) - \phi_h (M_h - B_h) - \phi_y \left(M_y - B_y\right) \\
& - C_{d\beta} M_{d\beta} - \lambda_{d\beta}\left[\left(\mu_u + \delta_r\right) R_{\alpha\beta} - \left(\mu_h - 20\right) M_{d\beta}\right] \\
& - C_{d\gamma} M_{d\gamma} - \lambda_{d\gamma}\left[\left(\mu_u + \delta_r\right) R_{\alpha\gamma} - \left(\mu_h - 20\right) M_{d\gamma}\right]
\end{aligned} \tag{17}
$$

Equation (17) represents only part of the programming model, it pertains just to region α, but it incorporates the transfer of effluent to regions β and γ. The regional indexes are added to expressions pertaining to transferring and treating water and effluent in regions β and γ ($\alpha\beta$ indexes effluent moved from α to β; $d\beta$ is for desalination in β). The expressions in last two lines of the equation are the cost of desalination region's β and region's γ water and the constraints specifying that all the salts brought with the effluent from α to β and γ are removed. We add (18), salt concentration in urban water in region α in the solution of (17), it is

(18) $$\mu_u = \frac{\mu_h M_h + \mu_y M_y}{M_h + M_y} = \frac{250*20.6+160*47.5}{20.6+47.5} = 187.22$$

We are interested in the marginal cost of the provision of water in region α. As an example, examine the first order condition of maximizing (17) with respect to Mountain water

(19)
$$\frac{\partial L_\alpha}{\partial M_y} = -C_y + \lambda_m - \lambda_d \mu_y + \lambda_d \left(R_s + R_{\alpha\beta} + R_{\alpha\gamma}\right)\mu'_u - \lambda_{d\beta} R_{\alpha\beta} \mu'_u$$
$$-\lambda_{d\gamma} R_{\alpha\gamma} \mu'_u - \phi_y = 0$$
$$\lambda_m = C_y + \lambda_d \mu_y - \lambda_d \left(R_s + R_{\alpha\beta} + R_{\alpha\gamma}\right)\mu'_u + \lambda_{d\beta} R_{\alpha\beta} \mu'_u + \lambda_{d\gamma} R_{\alpha\gamma} \mu'_u + \phi_y$$
$$\mu'_u \equiv \frac{\partial \mu_U}{\partial M_y} = \frac{\mu_y \left(M_h + M_y\right) - \left(\mu_h M_h + \mu_y M_y\right)}{\left(M_h + M_y\right)^2}$$

Table 3 reports the marginal cost for region α and their components. The marginal cost of fresh water, whether from the Coastal or the Mountain aquifer, is the dual value $\lambda_m = 2.815$; i.e. NIS 2.815 per CM (NIS 4.5=1 US dollar). The components of λ_m were calculated in the table using the program multipliers $\lambda_d = 0.0047, \lambda_{d\beta} = 0.0014, \lambda_{d\gamma} = 0.0039, \phi_y = 1.334$. With these magnitudes, the third and the second lines in (19) were calculated as

$$\mu'_u = \frac{160*(20.6+47.5) - (250*20.6+160*47.5)}{(20.6+47.5)^2} = -0.400$$

(20)
$$\lambda_m = 0.675 + .0047*160 - 0.0047*(29.6+4.7+6.8)*(-0.400)$$
$$+0.0014*29.6*(-0.400) + 0.0039*4.7*(-0.400) + 1.334$$
$$= 0.675 + 0.752 + 0.077 - 0.016 - 0.007 + 1.334 = 2.815$$

The last line in (20) is the Mountain line in Table 3. In considering these magnitudes recall that λ_m is the VMP of fresh water and it is also its marginal cost. For water from the Mountain aquifer, the cost of withdrawing and moving an additional CM to the Coast is NIS 0.675; the cost of removing the salts imported with one CM is NIS 0.752. Adding a CM of Mountain water changes the composition of the water supplied to town and hence the salt content of the effluent. These changes are reflected in the other three columns under Salt removal. The remainder is the scarcity value. The Coast line in the table was calculated similarly. There is no direct salt removal cost in the Coast since water withdrawn from the Coastal aquifer and used above it do not *add* salts to the local reservoir.

Table 3. Marginal Cost in Region α, Economic Alternative, NIS per CM.

	Marginal	Provision	Salt removal	Scarcity

	Cost		Direct	Export from α	Import to β	Import to γ	
Fresh							
Coast	2.815	0.450		-0.179	0.039	0.017	2.488
Mountain	2.815	0.675	0.752	0.077	-0.016	-0.007	1.334
Effluent	2.250	1.570					0.675

Note: The exchange rate for the table is NIS 4.5 per one US dollar.

As indicated in Table 2, in the agricultural alternative, effluent is both used in agriculture and discharged into the sea. Equation (13) can be derived unmodified from the first order conditions of (17). Calculating the two shadow prices in (13), we get (δ_r=100; $\mu_u+\delta_r$=287.22)

$$\lambda_{ra} = 2.25 - 1.575 = 0.675 \qquad (21)$$
$$\lambda_{rs} = -0.675 + 0.0047 * 287.22 = 0.675$$

The first line in (21) is the last row in Table 3. Since in Region α effluent is both used in agriculture and removed to the sea, the two net marginal contributions are equal.

Concluding Remarks

The analysis in the article was limited to the maintenance of a given salt content, but water and effluent are contaminated by a large number of factors from many different sources. In some cases the appropriate treatment of salts will reduce the prevalence of other pollutants. Health hazards have mostly to be taken care of when the sewage is treated, their treatment was reflected in the article (implicitly) in cost of recycling. Some sources of contamination are point specific, such as in gas stations or industrial plants. Such case can be seen, in the framework of the article, as belonging to autonomous sources; but mostly they will have to be treated individually.

The analysis abstracted from the structure of the water economy, as if there were a single supplier who draws, moves, and distributes water; collects sewage, treat it, and provides agriculture or removes it away from the region. Being responsible for the quantity and quality of the resources, the supplier is constraint to safe yields and is obliged to remove salts. But the economy could also be decentralized; in Israel, for example, one company operates the national grid and different entities provide municipal water and sewage services. The model developed in this article can determine transfer prices between the units of a decentralized water economy.

Sewage treatment and recycling are often locally conducted—a town supplying a neighboring agricultural cooperative with discharged effluent. In such cases prices may be negotiated and not necessarily reflect marginal cost. Still, marginal costs calculated in programming models may assist in reaching negotiated agreements.

Two issues to be watched in application come into mind: one is that the efficiency of effluent relative to freshwater; it may be affected by local conditions. The other issue is the setting of extraction levies. Private well owners, in the urban sector or in agriculture, may abandon their wells if the cost of own water—withdrawal *plus* levy—is set equal to water transferred from other regions. Where such abandonment is not desired, the levies will have to be set lower than indicated in programs.

References

Hillel, D. 2000. *Salinity Management for Sustainable Irrigation*. Washington D.C.: The World Bank.

Feinerman, E., Y. Plessner, and D. M. DiSegni Eshel. 2001. “Recycled Effluent: Should the Polluter Pay?” *American Journal of Agricultural Economics* 83:958-971.

Kislev, Y. 2006. “The Water Economy of Israel.” In: K. D. Hambright, F. Jamil, and J. Ginat eds. *Water in the Middle East*. Norton OK: University of Oklahoma Press, pp. 127-150.

Simon, C. P., and L. Blume. 1994. *Mathematics for Economists.* New York, London: W. W. Norton.

Appendix

Calibration of the Production Function in Equation (15)

The data available for the country’s 21 agricultural regions for 1999 were on three factors: irrigated land, labor, and water. The production function was formulated for region i as

(A.1) $$Y_i = KS_i^{0.40} D_i^{0.45} W_i^{0.15}$$

where Y_i is production in NIS per year, S_i irrigated land area, D_i labor in man-years, and W_i water in CM per year. The coefficients, 0.40, 0.45, and 0.15, are calculated factor shares for the sector. We added the assumption that the ratio of the contribution of effluent to productivity to that of freshwater is $\sigma = 0.80$ $\left(W = M + \sigma R\right)$. The value of production was available only for the sector as a whole, Y,

(A.2) $$Y = \sum_{i=1}^{21} KS_i^{0.40} D_i^{0.45} W_i^{0.15}$$

the value of K was solved from (A.2) $K = Y / \sum_{i=1}^{21} S_i^{0.40} D_i^{0.45} W_i^{0.15}$ and setting $A_i = KS_i^{0.4} D_i^{0.15}$, the regional production function is

(A.3) $$Y_i = A_i W_i^{0.15}$$

The calibrated production function was tested for 1999. The predicted value of production for the 21 regions of the country was 1% short of the actual value and the predicted water use was 3% short. Close to the actual magnitudes.