Dynamic regulation of nonpoint source pollution when the number of emitters is large

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Abstract

When a nonpoint source pollution process involves many polluters, each taking the aggregate pollution process parametrically, regulation policies based on strategic interactions among dischargers become ineffective. We offer a regulation mechanism for this case. The mechanism consists of inter-period and intra-period components. The first exploits ambient (aggregate) information to derive the optimal pollution process and the ensuing social price of emission. The intra-period mechanism implements the optimal output-abatement-emission allocation across the heterogenous, privately informed firms in each time period. The intra-period mechanism gives rise to the full information outcome when the social cost of transfers is nil. A positive social cost of transfers decreases both output and abatement in each time period, though the effect on emission is ambiguous.

Keywords: Repeated moral hazard, stock externality, Markov decision process.

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1 Introduction

Regulating nonpoint source pollution is complicated because of the inability to use individual effluent charges or quotas, and the bulk of the literature in this vein, following Segerson (1988), relies in one way or another on observed aggregate (ambient) indicators. The efficacy of ambient-based policies in attaining a desirable goal is based on their ability to exploit strategic interactions between individual dischargers. When the number of polluters is large enough to the extent that participants assume that their own contributions to aggregate pollution accumulation are negligible, strategic interactions vanish and ambient based policies become ineffective. This situation is common when emissions emanate from agricultural producers or in global pollution processes such as emissions of greenhouse gases (GHG).[1]

Aggregate emission is a byproduct of production and contributes to a pollution stock that inflicts damage. Emission can be reduced by costly abatement efforts. The profit-seeking firms operate in competitive spot markets, taking prices and the stock of pollution parametrically in each period of time. In particular, each firm assumes that its own contribution to aggregate emission, as well as its impact on other firms' decisions, is negligible. When individual emissions are observed, the problem reduces to that of regulating a stock externality, on which quite a bit is known (see, among others, Newell and Pizer 2003). The problem of regulating unobserved individual emissions when firms are privately informed with regard to their production efficiency and abatement efforts is considerably more involved; the dynamic regulation mechanism

[1] Agriculture and other land use sectors are major contributors to global GHG (Stern 2007, pp. 196-197) and typically consist of many, dispersed, heterogeneous producers. These are likely candidates for the nonpoint source pollution situation considered here.

developed here addresses this problem.

Like its static counterpart, the literature on dynamic regulation of nonpoint source pollution, initiated with Xepapadeas (1992), predominantly relies on ambient based policies applied to a group of strategically interacting polluters. Of main concern in this literature are properties of the assortment of equilibria obtained under different decision rules (of polluters) and contractual arrangements (between the regulator and the group of polluters), and ranking welfare outcomes vis-à-vis some social objective reference. Recent contributions along this line include Karp (2005) and Athanassoglou (2010).[2] This approach is of limited use in the present situation, where strategic interactions between polluters are absent.

The approach taken here is more closely related to the literature on repeated moral hazard, where a principal regulates many privately informed agents over time by contracting each agent separately. The dynamic nature of the principal-agent interactions induces strategic behavior on both parties and raises a host of contract issues, including the feasibility of implementing the optimal outcome via a series of spot contracts (see Chiappori et al. 1994). The principal-agent regulation mode has been used in a wide variety of market failure situations, including dynamic taxation of privately informed agents (see Kocherlakota 2005, and references therein). Here it is applied in the context of regulating a dynamic nonpoint source pollution process, where agents (firms) are privately informed with regard to their individual production efficiency and abatement efforts.

As in the dynamic taxation literature (Kocherlakota 2005), we distinguish

[2] A related line of literature deals with dynamic regulation of strategically interacting polluters when individual actions are observed (see Benchekroun and Long 1998, Wirl 2007, Harstad 2012, and references therein).

between two types of random shocks: aggregate and firm-specific (or idiosyncratic). The aggregate shocks affect the (ambient) pollution accumulation process due, e.g., to weather conditions such as wind, temperature and precipitation. The idiosyncratic shocks affect the performance (efficiency) of each firm in each period, due, e.g., to personnel change or new knowledge. The aggregate shocks are publicly observed; the idiosyncratic shocks are privately observed (by each firm) but publicly revealed via firms' observable outputs in each time period. Both production and abatement activities require no capital stocks, hence firms maximize profits in each time period. The only stock process in the model is that of pollution – the public bad.

The proposed mechanism addresses this situation by combining (i) an inter-temporal model that uses aggregate observations and available firms' information, and (ii) an intra-temporal mechanism based on a series of spot contracts between the regulator and each firm. These two components evolve together in time and interact with each other. The inter-temporal model receives from the intra-period mechanism the information needed to update the asymmetric information regarding firm's types (production efficiency) and determines the optimal stock of pollution, aggregate emission and the ensuing social price of emission. The latter varies over time with the pollution stock but is constant within a period, hence is independent of the intra-period emission flow.

Given the social price of emission, the intra-period mechanism implements the optimal output-abatement-emission allocation across the heterogenous, privately informed firms via spot contracts between the regulator and each firm. These contracts are based on (observable) outputs and exploit the property that the social price of emission, while changing over time with the stock of pollution, is constant within a time period and is thus independent of the (intra-period) emission flow. The implemented allocation is shown to be first-best (i.e., the same as that without asymmetric information) when the social cost of transfers is nil. When transfers entail social costs, firms' private information entails a rent and both outputs and abatements are smaller, though the effect on emission is ambiguous.

As noted above, the only stock in our model is that of pollution – the public bad. There are no private (firm level) capital stocks (e.g., abatement capital), hence firms are not forward looking. As a result, contracts can be specified for one period at a time (the intertemporal effect due to the pollution stock is incorporated via the social price of emission, obtained from the inter-period model). This property allows us to avoid a host of issues frequently encountered in dynamic contracts, such as hold-up problems, the feasibility of commitments and effects of renegotiation (see discussion in Harstad 2012). The lack of abatement capital is a simplification, but one that holds in many real world situations, including emission from agricultural production where abatement entails the use of biological methods (e.g., natural enemies) instead of chemicals or tillage practices that reduce CO2 release. The payoff is a sharper focus on the regulation of hidden individual actions (moral hazard) under asymmetric information (adverse selection) in a dynamic context.

The next section lays out the model's structure and assumptions. Sections 3 and 4 present, respectively, the inter-temporal and intra-temporal components of the mechanism and derive its desirable properties. Section 5 concludes and the appendix contains proofs.

2 Setup

Emission is generated by a large number (n) of firms as a byproduct of production and contributes to a pollution stock. The environmental damage is caused by the pollution stock. The firms operate in a competitive environment, maximizing profit in each time period while taking prices and the pollution stock parametrically. Emission affects environmental cost indirectly through its contribution to the stock of pollution. Firms' output-emission relation and costs of production and abatement are specified in Section 2.1. The observation-information structure is described in Section 2.2. The pollution-emission process is formulated in Section 2.3 and welfare is defined in 2.4. The regulation task is outlined in Section 2.5.

2.1 Firms: output, abatement and emission

Firm i's profit at time period t is $p(t)y_i(t) - C_i(y_i(t), \beta_i(t)) - a_i(t)$, where $p(t)$ is output price, taken parametrically by the firm, $y_i(t)$ is output, $C_i(\cdot, \cdot)$ is the production cost function, $\beta_i(t) \in [0, \bar{\beta}_i(t)]$ represents the firm's efficiency, referred to as type (the zero lower bound is assumed for convenience), and $a_i(t)$ is the abatement cost (effort). The firm's type $\beta_i(t)$ changes over time due to exogenous, firm-specific shocks (change of management, new information). The firm's subscript i and time argument t are suppressed when the discussion pertains to a particular (any) firm at a particular time period.

The cost function $C(\cdot, \cdot)$ is increasing and convex in output: $C_1(y, \beta) \equiv \partial C(y, \beta)/\partial y > 0$ and $C_{11}(y, \beta) \equiv \partial^2 C(y, \beta)/\partial y^2 > 0$. Since a higher β means a more efficient firm, both $C(y, \beta)$ and $C_1(y, \beta)$ decrease with β. Additional cost properties (including third derivatives) will be used. We summarize the properties of C (for all firms i at all time periods $t = 1, 2, ...$) in:

$$C_1 > 0,\ C_2 < 0,\ C_{11} > 0,\ C_{12} < 0,\ C_{111} \geq 0,\ C_{112} \leq 0,\ C_{122} \geq 0 \tag{2.1}$$

for all $y > 0$ and $\beta \in [0, \bar{\beta}]$, where subscripts 1 and 2 signify partial derivatives with respect to the first and second argument, respectively (e.g., $C_{12} \equiv \partial^2 C / \partial y \partial \beta$).

Emission is an unintended consequence (a byproduct) of production and depends, in addition to output, on abatement efforts (cost) $a_i(t)$ according to

$$e_i(t) = G_i(a_i(t)) y_i(t), \tag{2.2}$$

where $G_i(\cdot)$ is emission per unit output, representing abatement technology. The functions $G(\cdot)$ decrease at a diminishing rate with a nonnegative third derivative:

$$G'(a) < 0,\ G''(a) > 0,\ G'''(a) \geq 0,\ \ \forall\, a \in [0, \infty), \tag{2.3}$$

for all firms i (as above, the firm's subscript i and time argument t are suppressed when appropriate).

Firm i's output-abatement allocation in period t is denoted $g_i(t) = (y_i(t), a_i(t)) \in \mathbb{R}^2_+$ and $g(t) = (g_1(t), g_2(t), ..., g_n(t)) \in \mathbb{R}^{2n}_+$ represents output-abatement allocation of all firms. Aggregate emission at time period t equals

$$E(g(t)) = \sum_i e_i(t) = \sum_i G_i(a_i(t)) y_i(t). \tag{2.4}$$

2.2 Observations and information

Outputs $y_i(t)$ are observed in each time period, but neither abatement $a_i(t)$ nor emissions $e_i(t)$ are observed (by the regulator and by other firms).

Moreover, at the beginning of time period t, $\beta_i(t)$ is firm i's private information and is known to the regulator (and to other firms) up to a probability distribution: from the viewpoint of the regulator, the $\beta_i(t)$'s are independent random variates with distributions $F_{it} : [0, \bar{\beta}_i(t)] \mapsto [0, 1]$ and densities $f_{it}(\cdot) = F'_{it}(\cdot)$, $i = 1, 2, ..., n$, $t = 1, 2....$ We assume that, for all firms at all time periods, $f_{it}(b) > 0$ for all $b \in [0, \bar{\beta}_i(t)]$ and the hazard functions $h_{it}(b) = f_{it}(b)/[1 - F_{it}(b)]$ are nondecreasing.

The $\beta_i(t)$'s change over time as a result of idiosyncratic (firm-specific) shocks (due, e.g., to personnel change or new information) and is firm i's private information at the beginning of period t. At the end of period t, the observed output $y_i(t)$ identifies $\beta_i(t)$ via the profit maximization conditions induce by the mechanism rules (details are provided in Section 4). At the beginning of period t, the distribution of $\beta(t)$, $F_{it}(\cdot)$, depends on $\beta_i(t-1)$ but not on information before $t-1$ (i.e., from the viewpoint of the regulator, $\beta_i(t)$ is a Markov chain). At the initial period, all firm types posses well-defined prior distributions. Let $m_i(t) = m_i(\beta_i(t-1))$ denote the moments characterizing F_{it} and let $M(t) = (m_1(t), m_2(t), ..., m_n(t))$, $t = 1, 2, ...$, where $M(1)$, characterizing the initial type distributions, is given.

Given $M(t)$, the regulator can calculate the average (with respect to the β_i's) cost functions

$$\bar{C}_{it}(y) = \int_0^{\bar{\beta}_i(t)} C_i(y, b) f_{it}(b) db, \ i = 1, 2, ..., n, \tag{2.5}$$

at the beginning of time period $t = 1, 2, ...$

2.3 Pollution

Aggregate emission $E(g(t))$ contributes to a pollution stock $Q(t)$ according to

$$Q(t+1) = R(Q(t), E(g(t)), Z(t+1)),\ t = 1, 2, ... \tag{2.6}$$

where $Z(t)$, $t = 2, 3, ...$ are aggregate random terms representing stochastic effects such as wind, humidity, temperature and precipitation. An example of a pollution accumulation process is

$$R(Q, E, Z) = Q + [E - \delta Q]Z, \tag{2.7}$$

where $\delta \in [0, 1)$ is a parameter representing pollution decay and Z a unit-mean, nonnegative random variable. The support of Z may be discrete or continuous and the process $Z(t)$ is a Markov chain (i.e., the distribution of $Z(t)$ conditional on $\{Z(t-1), Z(t-2), ...\}$ is the same as its distribution conditional on $Z(t-1)$).

The pollution stock $Q(t)$ inflicts the damage $D(Q(t))$ at time period t, where $D : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is increasing and convex: $D'(\cdot) > 0,\ D''(\cdot) \geq 0$.

The pollution stock $Q(t)$ and the realization $Z(t)$ are observed at the beginning of period t. The functions $E(\cdot)$, $R(\cdot, \cdot, \cdot)$ and $D(\cdot)$ are known (the former, defined in (2.4), depends on the known abatement functions $G_i(\cdot)$).

2.4 Welfare

Period t's benefit equals[3]

$$\sum_i [py_i(t) - C_i(y_i(t), \beta_i(t)) - a_i(t)] - D(Q(t)).$$

[3]To simplify we assume a constant output price $p(t) = p$. The analysis extends to cases where output price depends on aggregate output or fluctuates around a mean that depends on aggregate output or when $p(t)$ is a Markov chain.

The expected benefit given the information available at the beginning of period t, i.e., given $M(t)$ characterizing F_{it}, $i = 1, 2, ..., n$, is

$$B(Q(t), g(t)) = \sum_i \left[p y_i(t) - \bar{C}_{it}(y_i(t)) - a_i(t) \right] - D(Q(t)), \qquad (2.8)$$

Given $Q(1)$ and $M(1)$, a production-abatement plan $\{g(t) \in \mathbb{R}_+^{2n},\ t = 1, 2, ...\}$ generates the (random) benefit flow $B(Q(t), g(t))$, $t = 1, 2, ...$ with the corresponding (random) present value

$$\sum_{t=1}^{\infty} \theta^{t-1} B(Q(t), g(t)),$$

where $\theta \in (0, 1)$ is the running (single period) discount factor. The present value is random due to the stochastic pollution process, defined in equation (2.6), and the stochastic evolution of firm types . Welfare is defined as the expected present value conditional on available information.

2.5 Regulation

The regulation mechanism consists of inter-period and intra-period models that progress together in time: the inter-period model determines the optimal pollution and aggregate emission processes; the intra-period mechanism implements the optimal policy vis-à-vis the heterogenous and privately informed firms. The inter-period task is performed using aggregate pollution-emission observations and firm's type distributions, and gives rise to the shadow price of pollution and the ensuing social price of emission.

The intra-period regulation is carried out by means of spot contracts defined in terms of observable output with each firm at each time period. These contracts implement the optimal output-abatement allocation corresponding to the social price of emission (determined by the inter-period model). The latter varies over time with the *stock* of pollution but is constant within a time period, independent of the aggregate emission *flow* during the period. The inter-period model is presented in the next section and the intra-period contracts are analyzed in Section 4.

3 Optimal pollution process and the social price of emission

We characterize the optimal pollution process and the ensuing social price of emission from a social planner (regulator) viewpoint. The sequence of events is as follows: (i) at the beginning of time period t, $Q(t)$ and $M(t)$ are observed; (ii) the output-abatement action $g(t) \in \mathcal{G} \subset \mathbb{R}_+^{2n}$ is taken, giving rise to $E(g(t))$ and $B(Q(t), g(t))$ (cf. equations (2.4) and (2.8)), where $\mathcal{G} \subset \mathbb{R}_+^{2n}$ is the set of feasible output-abatement allocations; (iii) $Z(t+1)$ is realized and observed, giving rise to $Q(t+1)$; $\beta_i(t)$, $\forall\, i$, is revealed by (observed) outputs (see Section 4), giving rise to $M(t+1)$; (iv) the process advances to period $t+1$ and so on.

The information available when the action $g(t)$ is taken is

$$\Omega(t) = \left\{ [Q(s), Z(s), g(s), \beta(s)]_{s=1}^{t-1}, Q(t), M(t) \right\},$$

where it is recalled that $g(s)$ identifies $E(g(s))$ (cf. equation (2.4)). A period t's decision rule, $d_t(\cdot)$, defines the action to be taken at time t given the available information: $g(t) = d_t(\Omega(t))$. A decision rule can be Randomized or Deterministic. A policy is a list of decision rules for each time period: $\{d_1(\cdot), d_2(\cdot), ...\}$. A decision rule is Markovian if it depends on the current state only: $g(t) = \psi_t(Q(t), M(t))$, where $\psi_t(\cdot)$ denotes a Markovian decision rule. A policy is Markovian if all its decision rules are Markovian and it is stationary if $\psi_t(\cdot) = \psi(\cdot)$ for all time periods. A stationary-Markovian policy is represented by ψ. We confine attention to Stationary-Markovian-Deterministic (SMD) policies and denote by Φ the set of all feasible Markovian-Deterministic decision rules (i.e, all $\psi(\cdot)$ such that $\psi(Q, M) \in \mathcal{G} \subset \mathbb{R}_+^{2n}$ for all $Q \in \mathbb{R}_+$ and feasible M vectors).[4] To simplify the notation, we suppress M as an argument (notice that the process $M(t)$ is an exogenous process and cannot be influenced by the regulator).

Given $Q(1) = Q$ and $M(1) = M$, the expected present value generated by an SMD policy ψ is

$$V^{\psi}(Q) = \mathbb{E}_t \sum_{t=1}^{\infty} \theta^{t-1} B(Q(t), \psi(Q(t)), \tag{3.1}$$

where $\mathbb{E}_t$ represents expectation conditional on information available at the beginning of period t. The optimal policy ψ^* satisfies

$$V^*(Q) \equiv V^{\psi^*}(Q) = \sup_{\psi \in \Phi} V^{\psi}(Q).$$

The value function $V^*(\cdot)$ satisfies the optimality equation

$$V^*(Q(t)) = \max_{g \in \mathcal{G}} \left\{ B(Q(t), g) + \theta \mathbb{E}_t V^*(R(Q(t), E(g), Z(t+1))) \right\} \tag{3.2}$$

and the optimal decision rule is defined by[5]

$$\psi^*(Q(t)) = \arg\max_{g \in \mathcal{G}} \left\{ B(Q(t), g) + \theta \mathbb{E}_t V^*(R(Q(t), E(g), Z(t+1))) \right\}. \tag{3.3}$$

In (3.2)-(3.3), $R(Q(t), E(g), Z(t+1))$ stands for $Q(t+1)$ (see equation (2.6)) and $E(g)$ is defined in (2.4).

[4]Because $Z(t)$ and $\beta(t)$ are Markov chains and the evolution process $R(\cdot)$ (equation (2.6)) and the single period reward $B(Q(t), g(t))$ (equation (2.8)) are autonomous, i.e., do not depend explicitly on time, the optimal value can be attained by an SMD policy (see Puterman 2005, Chapters 5-6).

[5]We assume that $V^*(Q)$ and $\psi^*(Q)$ exist and are differentiable (see conditions in Acemoglu 2009, p. 553).

Let the subscripts y_i and a_i denote partial derivatives with respect to the corresponding elements of g, where it is recalled that $g_i = (y_i, a_i)$ and $g = (g_1, g_2, ..., g_n)$. Necessary conditions for an interior optimum corresponding to (3.2) include $B_{y_i}(Q(t), \psi^*(Q(t))) - \tau(t)E_{y_i}(\psi^*(Q(t))) = 0$ or

$$p - C_{i1}(y_i^*(Q(t)), \beta_i(t)) - \tau(t)G_i(a_i^*(Q(t))) = 0, \ i = 1, 2, ..., n, \tag{3.4a}$$

and $B_{a_i}(Q(t), \psi^*(Q(t))) - \tau(t)E_{a_i}(\psi^*(Q(t))) = 0$ or

$$-1 - \tau(t)G_i'(a^*(Q(t)))y_i^*(Q(t)) = 0, \ i = 1, 2, ..., n, \tag{3.4b}$$

where

$$\tau(t) \equiv \tau(Q(t)) = -\frac{\partial}{\partial E}\mathbb{E}_t\{\theta V^*(R(Q(t), E(\psi^*(Q(t)), Z(t+1)))\} =$$
$$-\theta\mathbb{E}_t\{V^{*\prime}(R(Q(t), E(\psi^*(Q(t))), Z(t+1)))R_E(Q(t), E(\psi^*(Q(t))), Z(t+1))\} \tag{3.5}$$

In (3.5), $V^{*\prime}(Q) = \partial V^*(Q)/\partial Q$ is the shadow price of the pollution stock and $R_E(Q, E, Z) = \partial R(Q, E, Z))/\partial E$. Thus, $\tau(t)$ is the social price of emission, measuring the effect of a small (marginal) change in emission on the expected next-period value discounted to the current period. In (3.4a) and (3.4b), the "=" signs change to "$\leq$" at the corners of $y_i^*(Q(t)) = 0$ and $a_i^*(Q(t)) = 0$, respectively.

Equations (3.4) are the stochastic Euler equations corresponding to (3.2) and together with an appropriate transversality condition can be used to solve for the value and optimal decision rule functions $V^*(\cdot)$ and $\psi^*(\cdot)$.[6] The social price of emission, $\tau(t)$, can thus be calculated at the beginning of time period t, upon observing $Q(t)$ and $M(t)$.

[6]Puterman (2005) describes a host of algorithms for calculating these functions in actual practice (see example in Leizarowitz and Tsur 2012).

It is of interest to see the link between the social price of emission $\tau = \tau(Q)$ and the marginal damage of pollution $D'(Q)$. Use (3.2)-(3.3) to write

$$V^*(Q(t)) = B_t(Q(t), \psi_t^*(Q(t))) + \theta\mathbb{E}_t\{V^*(R(Q(t), E(\psi_t^*(Q(t))), Z(t+1)))\}$$

and differentiate with respect to $Q(t)$, invoking (3.4) and (2.8), to obtain (dropping the t and $t+1$ arguments for convenience)

$$V^{*\prime}(Q) = -D'(Q) + \theta\mathbb{E}\{V^{*\prime}(R(Q, E(\psi^*(Q)), Z))R_Q(Q, E(\psi^*(Q)), Z)\}, \quad (3.6)$$

where R_Q is the partial derivative of R with respect to Q. Consider the $R(\cdot)$ specified in (2.7) with the Z's nonnegative, unit-mean, i.i.d. variates. Suppose that the distribution of the optimal state process converges in the long run to a steady-state (stationary) distribution (see conditions in Stokey et al. 1989, Chapter 12). Let a double-bar over a variable indicate expectation under the steady state distribution, so that in the long run $\bar{\bar{Q}}(t+1) = \bar{\bar{Q}}(t) \equiv \bar{\bar{Q}}$, or $\bar{\bar{R}}(Q, E(\psi^*(Q)), Z) = \bar{\bar{Q}}$, implying (noting (2.7) and that $Z = Z(t+1)$ has a unit mean and is independent of $Q = Q(t)$) that $\bar{\bar{E}}(\psi_t^*(Q)) - \delta\bar{\bar{Q}} = 0$, i.e., in the long run emission equals pollution decay on average.

Now, evaluate (3.6) at $Q = \bar{\bar{Q}}$ and expand $R(\bar{\bar{Q}}, E(\psi_t^*(\bar{\bar{Q}})), Z)$ around its (long-run) mean $\bar{\bar{R}} = \bar{\bar{Q}}$ (recalling that $\mathbb{E}Z = 1$, so $\mathbb{E}R_Q = 1 - \delta$) to obtain

$$V^{*\prime}(\bar{\bar{Q}}) \approx -D'(\bar{\bar{Q}}) + \theta V^{*\prime}(\bar{\bar{Q}})(1-\delta),$$

which implies

$$-\theta V^{*\prime}(\bar{\bar{Q}}) \approx \frac{1}{r+\delta} D'(\bar{\bar{Q}}),$$

where r is the interest rate corresponding to θ, i.e., $\theta = \frac{1}{1+r}$. The above relation, noting Equation (3.5) and recalling $\mathbb{E}R_E = 1$, implies that in the long run $\tau(t)$ fluctuates around

$$\frac{1}{r+\delta} D'(\bar{\bar{Q}}).$$

When the stock of pollution does not depreciate, i.e., $\delta = 0$, increasing emission increase the pollution stock from now and forever. The additional pollution inflicts a constant flow of damage at the rate $D'(\bar{\bar{Q}})$ per time period. The price of emission is therefore the present value of the perpetual damage flow (at the time of emission) $D'(\bar{\bar{Q}})/r$. With pollution decay, $\delta > 0$, the present value is calculated based on the effective discount rate $r + \delta$.

4 The intra-period mechanism

The goal is to implement the optimal output-abatement allocation $g^*(t) = \psi^*(Q(t))$ across the n firms, using the social price of emission $\tau(t) = \tau(Q(t))$. If individual emissions were observed, an obvious way to proceed would have been to impose the Pigouvian tax $\tau(t)$ on individual emissions, forcing each firm to internalize its external effect. Unfortunately, this is impossible when individual emissions and abatements are unobserved and regulation thus resorts to the use of transfers. Notice that this problem persists also when the firms types (the β_i's) are common knowledge and it will prove useful to begin with this case.

4.1 Firm types are common knowledge

At the beginning of period t, given the pollution stock $Q(t)$ and the social price of emission $\tau(t)$, the regulator seeks to implement the optimal output-abatement allocation, $\psi^*(Q(t))$, across the n firms. This requires inducing firms to internalize the environmental damage $\tau(t)e_i(t) = \tau(t)G_i(a_i(t))y_i(t)$ they generate (recalling that neither $e_i(t)$ nor $a_i(t)$ are observed). To that end, the regulator issues firms transfers s_i, such that the post-transfer profit of firm i is (the time argument is suppressed when appropriate)

$$\pi_i = py_i - C_i(y_i, \beta_i) - a_i + s_i, \; i = 1, 2, ..., n. \tag{4.1}$$

The public cost associated with an output-abatement allocation (y_i, a_i), $i = 1, 2, ..., n$, consists of the cost of transfers plus the social cost of emission:

$$\sum_i \{s_i(1+\lambda) + \tau G_i(a_i)y_i\},$$

where $\lambda \in [0, \bar{\lambda}]$ is the social cost of transfer (i.e., a transfer of one dollar generates a deadweight loss of λ due, e.g., to transactions costs or distortions) and $\bar{\lambda}$ is a finite upper bound. Subtracting the public cost $\sum_i \{s_i(1+\lambda) + \tau G_i(a_i)y_i\}$ from the sum of post-transfer profits gives period t's benefit

$$\sum_i \{py_i - C_i(y_i, \beta_i) - a_i + s_i - \tau G_i(a_i)y_i - (1+\lambda)s_i\}$$

which, using (4.1), can be rendered as

$$\sum_i \{(1+\lambda)(py_i - C_i(y_i, \beta_i) - a_i) - \tau G_i(a_i, \beta_i)y_i - \lambda\pi_i\}. \tag{4.2}$$

The optimal y_i, a_i and s_i (or π_i) maximize (4.2) subject to the participation constraints $\pi_i \geq 0$ and nonnegativity of y_i and a_i. The structure of (4.2) implies that the maximization can be carried out for each firm separately and proceed in two steps: first, firm i's output-abatement allocation (y_i^*, a_i^*) that maximize

$$J_i(y_i, a_i) = (1+\lambda)[py_i - C(y_i, \beta_i) - a_i] - \tau G(a_i)y_i \tag{4.3}$$

is chosen; second, the optimal transfer is set under the participation constraint $\pi_i \geq 0$. The necessary conditions corresponding to the maximization of (4.3) are

$$p - C_{i1}(y_i, \beta_i) = \frac{\tau G_i(a_i)}{1+\lambda} \tag{4.4a}$$

and

$$-G_i'(a_i)y_i = \frac{1+\lambda}{\tau}, \tag{4.4b}$$

and the optimal transfer is set such that

$$\pi_i = 0. \tag{4.4c}$$

In (4.4a) and (4.4b), the "=" signs change to "≤" at the corners of $y_i = 0$ and $a_i = 0$, respectively. Notice that conditions (4.4a)-(4.4b) are the same as conditions (3.4a)-(3.4b) when $\lambda = 0$ (zero social costs of transfers).

Following (4.4b), define

$$q_i(a) \equiv \frac{1+\lambda}{-G_i'(a)\tau}\,. \tag{4.5}$$

Substituting q_i for y_i in (4.4a) gives the condition

$$C_{i1}(q_i(a_i), \beta_i) + \frac{\tau}{1+\lambda}G_i(a_i) = p\,. \tag{4.6}$$

Suppose

$$C_{i11}(q_i(0), \beta_i)q_i'(0) + \frac{\tau}{1+\lambda}G_i'(0) > 0 \tag{4.7}$$

for all $\beta_i \in [0, \bar{\beta}_i]$ and $\lambda \in [0, \bar{\lambda}]$. Using (2.1), (2.3) and (4.5), it can be verified that (4.7) implies[7]

$$C_{i11}(q(a), \beta_i)q_i'(a_i) + \frac{\tau}{1+\lambda}G_i'(a_i) > 0 \text{ for all } a_i \geq 0, \tag{4.8}$$

i.e., the left-hand side of (4.6) is monotonic in a_i. If, in addition, for any $\beta_i \in [0, \bar{\beta}_i]$ and $\lambda \in [0, \bar{\lambda}]$, there exists some finite $\bar{a}_i$ (possibly very large) such that

$$C_{i1}(q_i(0), \beta_i) + \frac{\tau G_i(0)}{1+\lambda} < p \textit{ and } C_{i1}(q(\bar{a}_i), \beta_i) + \frac{\tau G_i(\bar{a}_i)}{1+\lambda} > p, \tag{4.9}$$

[7]Use properties of C and G, noting, from (4.5) and (2.3), that $q'(\cdot) > 0$ and $q''(\cdot) > 0$.

then (4.6) admits a unique solution $a_i^* \in [0, \bar{a}_i]$. In this case, (4.4a)-(4.4b) admit a unique, positive solution (a_i^*, y_i^*) with $y_i^* = q_i(a_i^*)$. The associated transfers are then defined, noting (4.1) and (4.4c), by $s_i^* = C_i(y_i^*, \beta_i) + a_i^* - py_i^*$.

Sufficiency requires that $J_i(\cdot, \cdot)$, defined in (4.3), is concave at $(y_i^*, a_i{}^*)$, which follows from:

Lemma 1. *Given (2.1), (2.3) and (4.8), $J_i(\cdot, \cdot)$ is concave on the domain*

$$y_i \geq q_i(a_i), \ (y_i, a_i) \in \mathbb{R}_+^2. \tag{4.10}$$

The proof is given in Appendix A. Since (y^*, a^*) satisfies (4.10) (cf. (4.4b) and (4.5)), it also satisfies the sufficient condition.

We conclude that the allocation $g^* = (g_1^*, g_2^*, ..., g_n^*)$, where $g_i^* = (y_i^*, a_i^*)$, $i = 1, 2, ..., n$, maximizes (4.2) and is therefore the socially optimal output-abatement allocation at time period t. We summarize the above discussion in:

Proposition 1. *Under (2.1), (2.3), (4.8) and (4.9), equations (4.4a)-(4.4b) admit a unique, positive solution $g_i^* = (y_i^*, a_i^*)$ for each $i = 1, 2, ..., n$, The corresponding output-abatement allocation $g^* = (g_1^*, g_2^*, ..., g_n^*)$ is socially optimal at time period t, given the social price of emission $\tau = \tau(t)$.*

Recalling that conditions (3.4a)-(3.4b) and (4.4a)-(4.4b) are the same when $\lambda = 0$, we conclude that:

Corollary 1. *Under the conditions of Proposition 1: (i) conditions (3.4a)-(3.4b) admit a unique, positive solution $\psi_i^*(Q(t)) \equiv (y_i^*(Q(t)), a_i^*(Q(t))$, $i = 1, 2, ..., n$; (ii) $\psi_i^*(Q(t)) = (y_i^*, a_i^*)$, $i = 1, 2, ..., n$, when $\lambda = 0$.*

We proceed now to the case where β_i is firm i's private information.

4.2 Private firm types information

The social price of emission $\tau = \tau(t)$ is given (obtained from the inter-period model). We show in Appendix B that, as in the case of known firm types, the optimal output-abatement-emission allocation can be attained by regulating each firm separately. We thus consider the regulation of an individual (any) firm and drop the firm's subscript i.

The mechanism consists of transfer and abatement functions, $\hat{s}(\cdot)$ and $\hat{a}(\cdot)$, defined in terms of (observed) output, and proceeds along the following steps: (i) The regulator announces the functions $\hat{s}(\cdot)$ and $\hat{a}(\cdot)$; (ii) the firm chooses output y and abatement $\hat{a}(y)$; (iii) upon observing y, the regulator pays the transfer $\hat{s}(y)$ and reimburses the firm for the abatement $\hat{a}(y)$. The transfer $\hat{s}(\cdot)$ is so specified that the firm's output choice is socially optimal. Since output is observable, using $\hat{s}(\cdot)$ to affect the firm's output choice is straightforward. Implementing abatement via the $\hat{a}(\cdot)$ function is more subtle since abatement is unobserved. We return to this issue after specifying the mechanism and verifying its desirable properties.

4.2.1 Specification of $\hat{s}(\cdot)$ and $\hat{a}(\cdot)$

The derivation of the transfer function, $\hat{s}(\cdot)$, and the abatement function, $\hat{a}(\cdot)$, builds on the following Direct Revelation Mechanism: The regulator announces the functions $Y(\cdot)$, $A(\cdot)$ and $S(\cdot)$, following which the firm reports its type b. Upon receiving the report b, the regulator assigns the contract $\{Y(b), A(b), S(b)\}$, indicating that the firm produces $Y(b)$, spends $A(b)$ on abatement activities and receives the transfer $S(b)$.[8]

[8] In general, a firm's contract depends on the firm's own report and on the reports of all other firms (in which case the mechanism is stochastic, since from the viewpoint of each firm the other firms types are uncertain). We verify in Appendix B that the optimal outcome

The mechanism is truthful if the firm will (voluntarily) report its type honestly, i.e., $b = \beta$. The firm's payoff when it reports b is

$$\tilde{\Pi}(b, \beta) = pY(b) - C(Y(b), \beta) - A(b) + S(b). \tag{4.11}$$

Necessary condition for truth telling is $\tilde{\Pi}_1(\beta, \beta) \equiv \partial\tilde{\Pi}(b, \beta)/\partial b|_{b=\beta} = 0$ or

$$[p - C_1(Y(\beta), \beta)]Y'(\beta) - A'(\beta) + S'(\beta) = 0. \tag{4.12}$$

Given $C_{12} < 0$ (cf. (2.3)), the monotonicity condition

$$Y'(x) \geq 0 \; \forall \; x \in [0, \bar{\beta}] \tag{4.13}$$

is sufficient for truth telling.[9]

The firm's payoff under honest reporting is

$$\Pi(\beta) = pY(\beta) - C(Y(\beta), \beta) - A(\beta) + S(\beta). \tag{4.14}$$

Invoking (4.12),

$$\Pi'(\beta) = -C_2(Y(\beta), \beta). \tag{4.15}$$

can be attained by a deterministic contract, which depends only on the firm's own report.

[9]This can be shown as follows (Laffont and Tirole 1993, p. 121). Suppose $b \neq \beta$ yields a larger payoff:

$$\tilde{\Pi}(b, \beta) > \tilde{\Pi}(\beta, \beta) \Rightarrow \int_\beta^b \tilde{\Pi}_1(x, \beta)dx > 0,$$

which invoking the necessary condition, $\tilde{\Pi}_1(x, x) = 0 \; \forall x \in [0, \bar{\beta}]$, can be expressed as

$$\int_\beta^b [\tilde{\Pi}_1(x, \beta) - \tilde{\Pi}_1(x, x)]dx = \int_\beta^b \int_x^\beta \tilde{\Pi}_{12}(x, z)dzdx > 0.$$

Now, $\tilde{\Pi}_{12}(x, z) = -C_{12}(q(x), z)Y'(x)$ and $C_{12} \leq 0$. If $b > \beta$, then $x \geq \beta$ and the above inequality becomes

$$-\int_\beta^b \int_\beta^x \tilde{\Pi}_{12}(x, z)dzdx > 0 \Rightarrow \int_\beta^b \int_\beta^x C_{12}(Y(x), z)Y'(x)dzdx > 0,$$

which is impossible when $Y'(x) \geq 0 \; \forall x \in [0, \bar{\beta}]$, ruling out the possibility that $\tilde{\Pi}(b, \beta) > \tilde{\Pi}(\beta, \beta)$ for $b > \beta$. Likewise, when $b < \beta$, the inequality reads $-\int_b^\beta \int_x^\beta \tilde{\Pi}_{12}(x, z)dzdx = \int_b^\beta \int_x^\beta C_{12}(x, z)Y'(x)dzdx > 0$, which is again impossible when $Y'(x) \geq 0$, ruling out the possibility $b < \beta$.

Since $C_2 < 0$ (cf. (2.3)), $\Pi(\cdot)$ is increasing and requiring

$$\Pi(0) = 0 \tag{4.16}$$

ensures a nonnegative profit for all firm types.

Noting (4.2), the firm's contribution to expected intra-period welfare is

$$\int_0^{\bar{\beta}} \{(1+\lambda)[pY(b) - C(Y(b), b) - A(b)] - \tau G(A(b), b)Y(b) - \lambda \Pi(b)\} f(b)db \tag{4.17}$$

The regulator seeks the functions $Y(b)$, $A(b)$ and $\Pi(b)$ that maximize (4.17) subject to (4.13), (4.15) and (4.16).

Consider the problem of maximizing (4.17) subject to (4.15)-(4.16), ignoring the monotonicity constraint (4.13). This is a standard Optimal Control problem with two controls, Y and A, and one state, Π. Let $Y^*(b)$, $A^*(b)$ and $\Pi^*(b)$ denote the solution of this problem. We verify in Appendix C that $Y^*(b)$ and $A^*(b)$ satisfy

$$p - C_1(Y^*(b), b) = \frac{\tau G(A^*(b))}{1+\lambda} - \frac{\lambda}{1+\lambda} \frac{1 - F(b)}{f(b)} C_{21}(Y^*(b), b) \tag{4.18a}$$

and

$$-G'(A^*(b))Y^*(b) = \frac{1+\lambda}{\tau}. \tag{4.18b}$$

Using (4.15)-(4.16), we obtain

$$\Pi^*(b) = \int_0^b -C_2(Y^*(z), z)dz \tag{4.19}$$

and (4.14) then gives

$$S^*(b) = \Pi^*(b) - [pY^*(b) - C(Y^*(b), b) - A^*(b)]. \tag{4.20}$$

It turns out that $Y^*(\cdot)$, $A^*(\cdot)$ and $\Pi^*(\cdot)$ are also the optimal solutions for the problem of maximizing (4.17) subject to (4.15)-(4.16) and the monotonicity constraint (4.13). This follows from:

Lemma 2. *Under (2.1), (2.3) and (4.7), $Y^{*\prime}(b) > 0$ and $A^{*\prime}(b) > 0$ for all $b \in [0, \bar{\beta}]$.*

The proof is given in Appendix D.

The optimal output and abatement are, respectively,

$$y^{*\lambda} \equiv Y^*(\beta) \tag{4.21}$$

and

$$a^{*\lambda} \equiv A^*(\beta). \tag{4.22}$$

From (3.4a)-(3.4b), (4.4a)-(4.4b) and (4.18a)-(4.18b), we see that

$$(y_i^{*\lambda}, a_i^{*\lambda}) = (y_i^*, a_i^*) = \psi_i^*(Q(t)),\ i = 1, 2, ..., n,$$

when $\lambda = 0$ (zero social cost of transfers), where $\psi_i^*(Q(t))$ is the social planner output-abatement allocation of firm i, satisfying (3.4a)-(3.4b). A positive λ causes these allocations to deviate from each other. The wedge between $(y_i^{*\lambda}, a_i^{*\lambda})$ and (y_i^*, a_i^*) reflects the firm's information rent associated its type being a private information; it is represented by the rightmost term of equation (4.18a) evaluated at $b = \beta_i$. The wedge between (y_i^*, a_i^*) and $\psi_i^*(Q(t))$ is due to the fact that the former allocations are implemented when individual abatements and emissions are unobserved (via the transfers s_i^*), whereas the allocations $\psi_i^*(Q(t))$ are determined by the social planner from the outset. The allocation $(y_i^{*\lambda}, a_i^{*\lambda})$, $i = 1, 2, ..., n$, is optimal when firm types and abatements are private information and individual emissions are unobserved.

4.2.2 Implementation

With a monotonic $Y^*(\cdot)$, the inverse function $\varphi \equiv Y^{*-1} : \mathbb{R}_+ \mapsto [0, \bar{\beta}]$ exists, is increasing and satisfies, noting (4.21),

$$\varphi(y^{*\lambda}) = \beta. \tag{4.23}$$

Following (4.19), define

$$\hat{\pi}(y) = \int_{Y^*(0)}^{y} -C_2(z, \varphi(z))\varphi'(z)dz \tag{4.24}$$

for $y \geq Y^*(0)$. The functions $\hat{s}(\cdot)$ and $\hat{a}(\cdot)$ are defined by:

$$\hat{s}(y) \equiv \hat{\pi}(y) - [py - C(y, \varphi(y))] \tag{4.25}$$

and

$$\hat{a}(y) \equiv A^*(\varphi(y)). \tag{4.26}$$

The contract consisting of the transfer and abatement functions specified in (4.25)-(4.26) is called the $[\hat{s}, \hat{a}]$ contract. We show that:

Proposition 2. *The $[\hat{s}, \hat{a}]$ contract implements the optimal output-abatement allocation $(y^{*\lambda}, a^{*\lambda})$.*

Proof. Noting (4.25), the firm's post-transfer profit, $py - C(y, \beta) + \hat{s}(y)$, equals

$$C(y, \varphi(y)) - C(y, \beta) + \hat{\pi}(y).$$

The profit maximizing output satisfies, noting (4.24),

$$C_1(y, \varphi(y)) - C_1(y, \beta) = C_{12}(y, \tilde{\beta})[\varphi(y) - \beta] = 0$$

for some $\tilde{\beta}$ between β and $\varphi(y)$. Since $C_{12}(y, \cdot) < 0$ and $\varphi(\cdot)$ is increasing, $y^{*\lambda}$ (cf. (4.21)) is the unique profit maximizing output, implying that the transfer $\hat{s}(\cdot)$ implements the optimal output $y^{*\lambda}$.

Noting (4.23), the output $y^{*\lambda}$ identifies β, which together with (4.22) and (4.26) implies $\hat{a}(y^{*\lambda}) = a^{*\lambda}$, giving rise to the optimal abatement. $\square$

We reiterate the final point of the proof in:

Remark 1. *The observed output of firm i reveals its type β_i (cf. equation (4.23)).*

The firm types $\beta_i(t)$, $i = 1, 2, ..., n$, give rise to $m_i(t+1)$, identifying the distribution of $\beta_i(t+1)$ at the beginning of period $t+1$, to be used at the beginning of period $t+1$ by the inter-period model.

As was noted above, implementing the optimal output via $\hat{s}(\cdot)$ is straightforward since output is observable. Implementing the abatement via $\hat{a}(\cdot)$ is more subtle since abatement is unobserved. How can the regulator verify that the firm actually carries out the abatement $\hat{a}(y^{*\lambda})$ when he cannot observe abatement efforts in actual practice? After all, receiving an abatement subsidy and performing abatement activities are two different things: the first is mutually observed while the second is known only to the firm. This problem is resolved when the regulator observes total cost $C + a$. This is so because the firm's output choice reveals the firm's type β (Remark 1), hence ex post (after y has been observed and the true type β revealed) the regulator can calculate the production cost $C(y^{*\lambda}, \beta)$ and subtract from the total cost $C + a$ to obtain the abatement cost.

The abatement effort (cost) in our model is similar to the cost reduction effort in Laffont and Tirole (1986) and the use of cost reimbursement is therefore similar. While in Laffont and Tirole (1986) the entire firm's cost is reimbursed, here only part of the cost – that due to abatement – is reimbursed. In our case, if the total cost were observed, the regulator could infer the firm's abatement cost and reimburse it accordingly.

If total cost is unobserved, some extraneous device is needed to deter abatement shirking, i.e., to ensure that firms carry out the abatement activities for which they are reimbursed. For example, a monitoring-sanction scheme enforced by the court system (see Shavell 1987).

When $\lambda = 0$ (zero social cost of transfers), the $[\hat{s}, \hat{a}]$ contracts implement the first-best allocation $(y_i^*, a_i^*) = \psi_i^*(Q(t))$ for all i. To see this, note that $y^{*\lambda} = Y^*(\beta)$ and $a^{*\lambda} = A^*(\beta)$, where $Y^*(\beta)$ and $A^*(\beta)$ solve (4.18a)-(4.18b) with $b = \beta$. But when $\lambda = 0$, (4.18a) is the same as (4.4a) and (4.18b) is the same as (4.4b). Since the solution of (4.4a)-(4.4b) is unique (Proposition 1), the two solutions must be the same (the equality to ψ_i^* follows from Corollary 1). Under zero social cost of transfers, the regulator can nullify the firm's information rent and the optimal regulations attains the first-best outcome.

When $\lambda > 0$, (4.18a) implies (recalling $C_{12} < 0$ and dropping the firm's subscript i),

$$p - C_1(q(a^{*\lambda}), \beta) - \frac{\tau G(a^{*\lambda})}{1+\lambda} > 0$$

where, $q(a) = -(1+\lambda)/(\tau G(a))$ is defined in (4.5). Likewise, from (4.4a),

$$p - C_1(q(a^*), \beta) - \frac{\tau G(a^*)}{1+\lambda} = 0.$$

Subtracting the latter from the former gives

$$C_1(q(a^*), \beta) - C_1(q(a^{*\lambda}), \beta) + \frac{\tau}{1+\lambda}\left[G(a^*) - G(a^{*\lambda})\right] > 0.$$

The above inequality can be expressed as

$$\int_{a^{*\lambda}}^{a^*} \left[C_{11}(q(\alpha), \beta) q_1(\alpha) + \frac{\tau}{1+\lambda} G'(\alpha)\right] d\alpha > 0.$$

In view of (4.8), the integrand (the term inside the square brackets) is positive, implying $a^{*\lambda} < a^*$, hence $y^{*\lambda} = q(a^{*\lambda}) < q(a^*) = y^*$. We summarize the above discussion in:

Proposition 3. *(i) When $\lambda = 0$ (zero social cost of transfers), the $[\hat{s}, \hat{a}]$ mechanism implements the first-best allocation: $(y^{*\lambda}, a^{*\lambda}) = (y^*, a^*)$. (ii) When $\lambda > 0$, the mechanism gives rise to smaller output and abatement: $y^{*\lambda} < y^*$ and $a^{*\lambda} < a^*$.*

When transfers are costly, noting that $G(\cdot)$ is decreasing, emission, $G(a^{*\lambda})y^{*\lambda}$, may exceed or fall short of $G(a^*)y^*$, depending on the specifications of the underlying production and abatement technologies and the asymmetric information (type distribution).

4.3 Discussion

The regulation proceeds along the following stages. At the beginning of period t, $Q(t)$ and $M(t)$ are observed, based on which the social price of emission, $\tau(t)$ is calculated (Section 3). Based on $\tau(t)$ and the distributions of firm types (characterized by $M(t)$), an $[\hat{s}, \hat{a}]$ contract is specified and applied for each firm (Section 4.2). Upon observing outputs, abatements are reimbursed according to $\hat{a}(y)$ (the $\hat{a}$ part of the $[\hat{s}, \hat{a}]$ contract – equation (4.26)) and the firm types $\beta_i(t)$, $i = 1, 2, ..., n$, are revealed (Remark 1), giving rise to $M(t+1)$ (Section 2.2). The output-abatement allocation gives rise to aggregate emission $E(g(t))$ (equation (2.4)). A realization of $Z(t+1)$ forms $Q(t+1)$ (equation (2.6)). These steps are repeated in period $t+1$. The mechanism is initiated given $Q(1)$ and $M(1)$.

Noting Lemma 2, the mechanism rewards efficiency in that output increases with the firm's type. Since more output implies more emission (given abatement), the mechanism also requires that more efficient firms (with a higher β_i) will spend more on abatement (Lemma 2 again).

The regulation budget in period t is $\sum_i \hat{s}_i(y_i(t)) + \sum_i \hat{a}_i(y_i(t))$. The first term is the total transfer needed to induce firms to account for their external effects in their production decisions. The second term is the total cost of abatement. Covering the cost of abatement from public funds is justifiable since abatement activities benefit the public at large. The transfer cost (the first term) is due to the nonpoint source pollution feature (the unobserved individual emissions) and using public funds to finance it may be controversial. Regulation budget considerations raise a whole range of issues that lie outside the present scope (see discussion in Hyde et al. 2000). We only note that incorporating lump sum transfers within the firms contracts, e.g., by subtracting a constant amount from the $\hat{s}(y)$ part of each $[\hat{s}, \hat{a}]$ contract, will not affect the resulting allocation but will affect the regulation budget in each period.

5 Concluding comments

The literature on dynamic regulating of nonpoint source pollution processes relies on ambient-based policies applied to strategically interacting polluters. Such policies are of limited use when polluters assume that their own contributions to ambient (aggregate) pollution as well as their effect on other polluters decisions are negligible, which is often the case when emitters are numerous and dispersed. We offer a dynamic regulation mechanism for such situations. The mechanism consists of inter-period and intra-period components that evolve together in time and interact with each other. The inter-period model calculates the optimal pollution process and the ensuing social price of emission in each time period, based on aggregate (ambient) pollution observations and available firms' information (type distributions). Given the emission price, the intra-period mechanism implements the optimal output-abatement allocation across the heterogenous, privately informed firms in each time period via spot contracts, designed for each firm separately.

A firm's contract consists of a transfer function and an abatement function, both defined in terms of the firm's (observable) output. The transfer function induces the firm to internalize its external effect. Given the output choice, the abatement function determines the optimal abatement effort. The firm's output choice resolves the asymmetric information and allows implementation of optimal abatement when the firm's total cost is observed. If total cost is not observed, an additional device is needed to ensure that the firm actually carries out the abatement for which it has been reimbursed, e.g., a monitoring-sanction scheme enforced via the court system.

Given the social price of emission (determined by the inter-period model), the intra-period contracts implement the first-best output-abatement allocation when the social cost of transfers is nil. When the social cost of transfers is positive, the optimal output and abatement, implemented by the mechanism, are smaller than their complete information counterparts, though emission may be larger or smaller (less abatement increases emission while smaller output decreases emission).

The ability to implement the intra-period output-abatement allocations via a series of spot contracts owes to the lack of private (firm specific) stocks (e.g., abatement capital): the only stock variable in the model is the (public) stock of pollution. Extending the present framework to account for private capital stocks remains a challenge for future research.

Appendix

A Proof of Lemma 1

Noting that $J_{yy} \equiv \partial^2 J/\partial y^2 < 0$ and $J_{aa} \equiv \partial^2 J/\partial a^2 < 0$, we need to show that the determinant of the Hessian matrix of $J(y, a)$,

$$H_J = (1+\lambda)C_{11}(y,\beta)\tau G''(a)y - \tau^2 G'^2(a),$$

is nonnegative. Noting $C_{111} \geq 0$ (cf. (2.1)) and (4.10)

$$H_J \geq (1+\lambda)C_{11}(q(a),\beta)\tau G''(a)q(a) - \tau^2 G'^2(a),$$

so we need to show that the term on the right-hand side above is nonnegative or, alternatively, that

$$\frac{1+\lambda}{\tau}\frac{1}{-G'(a)}C_{11}(q(a),\beta)G''(a)q(a) + G'(a) \geq 0. \tag{A.1}$$

Noting (4.5), (4.10) and $q'(a) = \frac{1+\lambda}{\tau}G''(a)/G'^2(a)$, the left-hand side of (A.1) becomes

$$C_{11}(q(a),\beta)q'(a)[-G'(a)q(a)] + G'(a).$$

Since $-G'(a)q(a) = (1+\lambda)/\tau$ (cf. (4.5)), this expression can be rendered as

$$C_{11}(q(a),\beta)q'(a) + \frac{\tau}{1+\lambda}G'(a),$$

which is nonnegative by (4.8), implying that inequality (A.1) holds.

B Optimality of deterministic mechanisms

In general, contracts are specified in terms of functions that depend on the reports of all firms (mechanisms based on such contracts are stochastic, since from the viewpoint of a single firm, the other firms types are uncertain). We verify that the maximal expected welfare can be attained by a deterministic mechanism, where contracts are specified for each firm separately and depend only on the firm's own report.

Denote by B_{-i} the vector of the true types of all firms except firm i. The mechanism is truthful if firm i will (voluntarily) report its type honestly, i.e., $b_i = \beta_i$, when all other firms report honestly. Firm i's expected payoff when it reports b_i and all other firms report their true types is

$$\pi_i(b_i, \beta_i) = E_{B_{-i}}\{py_i(b_i, B_{-i}) - C_i(y_i(b_i, B_{-i}), \beta_i) - a_i(b_i, B_{-i}) + s_i(b_i, B_{-i})\}. \tag{B.1}$$

The firm will report honestly if $\pi_i(\beta_i, \beta_i) \geq \pi_i(b_i, \beta_i)\ \forall\, b_i \in [0, \bar{\beta_i}]$. The necessary condition for truthtelling is $\pi_{i1}(\beta_i, \beta_i) \equiv \partial\pi_i(b_i, \beta_i)/\partial b_i|_{b_i=\beta_i} = 0$ or

$$E_{B_{-i}}\{[p - C_{i1}(y_i(\beta_i, B_{-i}), \beta_i)]y_{i1}(\beta_i, B_{-i}) - a_{i1}(\beta_i, B_{-i}) + s_{i1}(\beta_i, B_{-i})\} = 0. \tag{B.2}$$

Firm i's payoff under honest reporting is

$$\tilde{\pi}_i(\beta_i) = E_{B_{-i}}\{py_i(\beta_i, B_{-i}) - C_i(y_i(\beta_i, B_{-i}), \beta_i) - a_i(\beta_i, B_{-i}) + s_i(\beta_i, B_{-i})\}. \tag{B.3}$$

Differentiating with respect to β_i and invoking (B.2) gives

$$\tilde{\pi}_i'(\beta_i) = E_{B_{-i}}\{-C_{i2}(y_i(\beta_i, B_{-i}), \beta_i)\}. \tag{B.4}$$

Since $C_{i2} < 0$ (cf. (2.1)), $\tilde{\pi}_i(\cdot)$ is increasing and requiring

$$\tilde{\pi}_i(0) = 0 \tag{B.5}$$

ensures a nonnegative profit for all types.

Period t's welfare (4.2) generalizes to

$$v = \sum_i E_{\beta_i}\{E_{B_{-i}}\{J_i(y_i(\beta_i, B_{-i}), a_i(\beta_i, B_{-i}))\} - \lambda\tilde{\pi}_i(\beta_i)\} \tag{B.6}$$

where $J_i(y_i, a_i)$ is defined in (4.3). The regulator seeks the functions $y_i(\cdot,\cdot)$, $a_i(\cdot,\cdot)$ and $\tilde{\pi}_i(\cdot)$ that maximize v subject to (B.4) and (B.5). Let v^* be the optimal expected welfare, i.e., the value (B.6) evaluated at the optimal mechanism. Then,

Proposition 4. *Under (2.1), (2.3) and (4.8), v^* can be realized by deterministic contracts $\{Y_i(\cdot), A_i(\cdot), S_i(\cdot)\}$, each depending on firm i's own report.*

Proof. We begin by showing that the optimal mechanism satisfies (4.10), i.e.,

$$y_i(\beta_i, B_{-i}) \geq q(a_i(\beta_i, B_{-i}))\;\forall\, i, \tag{B.7}$$

where $q(\cdot)$ is defined in (4.5). Suppose otherwise, that $y_i < q(a_i)$. Then (recalling $J_a(q,a) = 0$, $J_{aa} < 0$ and $q' > 0$), as long as $y_i < q(a_i)$, decreasing a_i (keeping y_i constant) increases J_i without any effect on $\tilde{\pi}_i$ (which depends on y_i via (B.4)-(B.5)), thereby increasing the term inside $E_{\beta_i}\{\cdot\}$ in (B.6) and the ensuing value, which cannot be optimal. We thus confine attention to the domain $(y_i, a_i) \in \mathbb{R}_+^2$ satisfying (B.7) (or (4.10)), over which (Lemma 1) $J_i(y_i, a_i)$ is concave.

We can now show that to any stochastic mechanism there corresponds a deterministic mechanism that performs at lease as well, in that it generates an expected welfare which is at least as large as that generated by the underlying stochastic mechanism. Let $Y_i(\beta_i) \equiv E_{B_{-i}}\{y_i(\beta_i, B_{-i})\}$ and $A_i(\beta_i) \equiv E_{B_{-i}}\{a_i(\beta_i, B_{-i})\}$. Then, using the concavity of $J_i(y,a)$, we obtain (Jensen's inequality),

$$E_{B_{-i}} J_i(y_i(\beta_i, B_{-i}), a_i(\beta_i, B_{-i})) \leq J_i(Y_i(\beta_i), A_i(\beta_i)).$$

Moreover, $C_{211} \leq 0$ (cf. (2.1)) implies $E_{B_{-i}}\{-C_{i2}(y_i(\beta_i, B_{-i}), \beta_i)\} \geq -C_{i2}(Y_i(\beta_i), \beta_i)$

for all $\beta_i \in [0, \bar{\beta}_i]$, hence

$$\tilde{\pi}_i(\beta_i) = \int_0^{\beta_i} E_{B_{-i}}\{-C_{i2}(y_i(x, B_{-i}), x)\}dx \geq \int_0^{\beta_i} -C_{i2}(Y_i(x), x)dx = \Pi_i(\beta_i),$$

where Π_i is obtained from $\Pi_i'(\beta_i) = -C_{i2}(Y_i(\beta_i), \beta_i)$ and $\Pi_i(0) = 0$. It follows that the expected welfare (B.6) corresponding to the deterministic mechanism $(Y_i(\cdot), A_i(\cdot), S_i(\cdot))$, where $S_i(\cdot)$ is derived from $\Pi_i(\cdot)$ according to (4.20), is at least as large as that obtained under the underlying stochastic mechanism. □

C Derivation of $Y^*(\cdot)$ and $A^*(\cdot)$

With $\mu(b)$ representing the costate variable, the Hamiltonian corresponding to the subproblem of maximizing (4.17) subject to (4.15)-(4.16) is

$$\mathcal{H}(b) = \{(1+\lambda)[pY(b) - C(Y(b), b) - A(b)] - \tau G(A(b))Y(b) - \lambda\Pi(b)\}f(b) \\ - \mu(b)C_2(Y(b), b).$$

Necessary conditions for an interior optimum include

$$\{(1+\lambda)[p - C_1(Y^*(b), b)] - \tau G(A^*(b))\} f(b) - \mu(b)C_{21}(Y^*(b), b) = 0, \quad \text{(C.1)}$$

$$-G'(A^*(b))Y^*(b) = \frac{1+\lambda}{\tau}, \quad \text{(C.2)}$$

$$\mu'(b) = \lambda f(b) \quad \text{(C.3)}$$

and the transversality condition, associated with free $\Pi(\bar{\beta})$,

$$\mu(\bar{\beta}) = 0. \quad \text{(C.4)}$$

Integrating (C.3), using (C.4), gives

$$-\mu(b) = \lambda[1 - F(b)]. \tag{C.5}$$

Substituting (C.5) in (C.1) and rearranging gives (4.18a) and (C.2) gives (4.18b).

D Proof of Lemma 2

Total differentiate (4.18b) gives

$$A^{*\prime} = \frac{-G' Y^{*\prime}}{G'' Y^*} \tag{D.1}$$

and totally differentiate (4.18a), using (D.1), gives $Y^{*\prime} M_1 = M_2$, where

$$M_1 = -C_{11} + \frac{\tau}{1+\lambda} \frac{G'^2}{G''} \frac{1}{Y^*} + \frac{\lambda}{(1+\lambda)h} C_{211},$$

and

$$M_2 = C_{12} \left(1 + \frac{\lambda}{1+\lambda} \frac{h'}{h^2} \right) - \frac{\lambda}{(1+\lambda)h} C_{212}$$

(the arguments $Y^*(b)$, $A^*(b)$ and b are suppressed for convenience). The non-decreasing hazard ($h' \geq 0$) together with (2.1) and (2.3) imply that $M_2 < 0$. We show that $M_1 < 0$.

Noting (2.1), the right-most term of M_1 is non-positive, so we need to show

$$-C_{11} + \frac{\tau}{1+\lambda} \frac{G'^2}{G''} \frac{1}{Y^*} < 0. \tag{D.2}$$

Recalling (4.5), multiply (D.2) by

$$q' = \frac{1+\lambda}{\tau} \frac{G''}{G'^2} > 0$$

to obtain

$$-C_{11} q' + \frac{1}{Y^*} < 0.$$

Invoking (C.2), the left-hand side above can be expressed as

$$-C_{11}q' - \frac{\tau}{1+\lambda}G', \tag{D.3}$$

which equals the negative of the left-hand side of (4.8) evaluated at $a = A^*$ and $\beta = b$, verifying inequality (D.2) and, thereby, $Y^{*\prime} > 0$. $A^{*\prime} > 0$, then, follows from (2.3) and (D.1).

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