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ENDOGENOUS RECOMBINANT GROWTH

by

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Endogenous Recombinant Growth

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Abstract

We extend Weitzman's (1998) recombinant growth framework to include endogenous R&D decisions. The analysis is carried out in the (knowledge-capital) state space by means of two characteristic curves: one is identified as a turnpike along which growing economies evolve; the other attracts stagnating economies. Sustained growth depends on a condition relating the slopes of the characteristic curves as well as on a minimal endowment requirement. A growing economy reaches the turnpike at a most rapid R&D rate and evolves along it thereafter. In the long run, the rate of growth and the income shares devoted to R&D, saving and consumption approach constant values that depend on the asymptotic characteristic slopes.

Keywords: knowledge generation, combined ideas, endogenous R&D, balanced growth

JEL Classification: C61, O31, O41

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1 Introduction

As technological progress is driven by assets loosely labelled as knowledge or human capital, understanding economic growth requires unfolding the mechanism through which these assets are accumulated. The endogenous growth literature predominately follows the original works of Romer (1986, 1990), Lucas (1988), Grossman and Helpman (1991) and Aghion and Howitt (1992).[1] In these studies the (sought-after) balanced long-run growth is obtained by imposing certain restrictions on the knowledge generation mechanism (Solow 2000). Weitzman's (1998) recombinant mechanism addresses these issues. In this mechanism, existing ideas are combined to generate new ideas. The number of new combinations is a combinatorial function of the number of existing ideas, and if this number were the only limiting factor in knowledge production, the model would give rise to an unrealistic super-exponential growth. Turning a potentially fruitful idea into useful knowledge, however, requires R&D efforts that consume resources. Weitzman (1998) assumes that a constant (exogenous) share of output is allocated to R&D and obtains balanced long-run growth. The limit to growth in this model stems not from lack of new ideas but from the limits on R&D resources that can be devoted to turn new ideas into useful knowledge.

In this work we extend Weitzman's (1998) recombinant growth framework to allow for endogenous R&D decisions. We provide a complete dynamic characterization and derive the conditions that give rise to sustained growth. The analysis is carried out in the (knowledge-capital) state space in terms of two characteristic curves, of which one is identified as a turnpike along which growing economies evolve and the other attracts stagnating economies. The turnpike approaches a straight

[1]For recent literature see Barro and Sala-i-Martin (2004) and Helpman (2004).

line at large knowledge levels while the stagnation curve turns out to be linear at all knowledge levels. A necessary growth condition is specified in terms of the slopes of these characteristic lines. Qualifying economies have the capacity to grow but need sufficient endowment to realize their growth potential. When the growth conditions are met, the knowledge-capital processes reach the turnpike at a most rapid R&D rate and evolve along it thereafter, eventually growing at a constant rate and devoting constant shares of income to R&D, saving and consumption.

Our method of analysis applies to situations in which investment in knowledge (or human capital) generation consumes a share of income and has been used to study other endogenous growth models (Tsur and Zemel 2004, 2005). As such knowledge investment rules are not uncommon (see, e.g., Shell 1967 and Chapter 5 of Barro and Sala-i-Martin 2004), the analysis extends beyond the present case of recombinant growth.

The next section revisits Weitzman's (1998) recombinant framework and formulates it in continuous time with endogenous R&D decisions. Section 3 motivates the endogenous formulation as the outcome of a competitive economy with knowledge as a regulated public good. Section 4 provides the dynamic characterization and establishes the properties of balanced growth. Section 5 presents an example illustrating the general results for a Cobb-Douglas economy. Section 6 concludes and an appendix contains technical derivations and proofs.

2 Formulation

Weitzman's (1998) knowledge generation mechanism is driven by the combinatorial power of hybridizing existing ideas to generate progeny ideas, which are then recombined with new and old ideas to generate yet newer ideas and so on. In Weitzman's (1998) notation, $A(t)$ stands for the stock of knowledge (useful ideas) at time t, $C_m(A)$ is the number of different combinations (or hybrids) of m elements of A (e.g., $C_2(A) = A(A-1)/2$) and

$$H(t) = C_m(A(t)) - C_m(A(t-1))$$

is the number of new m-combinations (or seed ideas) formed during time period t.

In a continuous time formulation we treat A, C_m and H as continuous functions and relate the rate at which new seed ideas are generated to the rate of change of knowledge stock according to

$$H(t) = C'_m(A(t))\dot{A}(t). \tag{2.1}$$

In fact, not every seed idea contributes to the state of knowledge. The probability π of a seed idea turning into useful knowledge depends on R&D efforts devoted for this purpose. According to Weitzman (1998), a fraction s of net output Y is invested in R&D efforts to hybridize H seed ideas. This R&D expenditure generates $H\pi(sY/H)$ useful new ideas that contribute to the knowledge base A. Knowledge, thus, evolves according to

$$\dot{A}(t) = H(t)\pi(s(t)Y(t)/H(t)). \tag{2.2}$$

The success probability π satisfies $\pi(0) = 0, \pi' > 0, \pi'' \leq 0$ and $\pi(\infty) \leq 1$.

Combining (2.1) and (2.2) gives

$$\dot{A}(t) = \frac{s(t)Y(t)}{\Phi(A(t))}, \tag{2.3}$$

where

$$\Phi(A) = C'_m(A)\pi^{-1}\left(\frac{1}{C'_m(A)}\right) \tag{2.4}$$

represents the expected unit cost of knowledge production. The function $\Phi(A)$ is non-increasing and the limit

$$\lim_{A\to\infty} \Phi(A) = 1/\pi'(0) > 0 \tag{2.5}$$

measures the expected unit cost when seed ideas abound.

The knowledge production process (2.3) is incorporated within the standard neoclassical framework, in which output is produced by capital and knowledge-augmented labor according to

$$Y(t) = F(K(t), A(t)L), \tag{2.6}$$

where F is concave and linearly homogenous with $F_K = \partial F/\partial K$ $(= F_1)$ and $F_A = \partial F/\partial A$ $(= F_2 L)$ denoting the marginal productivity of capital and knowledge, respectively, and $F_{11} < 0$, $F_{22} < 0$ and $F_{12} \geq 0$. To focus attention on endogenous growth, we assume that labor L is constant.

Our departure from Weitzman (1998) concerns the shares of income devoted to R&D and saving. Weitzman (1998) takes the R&D and saving shares as behavioral parameters, while here they are determined endogenously. Given the R&D share $s(t)$, capital evolves according to

$$\dot{K}(t) = (1 - s(t))Y(t) - Lc(t), \tag{2.7}$$

where $c(t)$ is per capita consumption at time t, generating the instantaneous utility $u(c(t))$ for some increasing and strictly concave function $u(c)$. The endogenous R&D-saving-consumption policy is the outcome of

$$V(K_0, A_0) = \max_{\{c(t),s(t)\}} \int_0^\infty Lu(c(t))e^{-\rho t} dt \tag{2.8}$$

subject to (2.3)-(2.7), $0 \leq s(t) \leq 1$, and other feasibility (e.g., non-negativity) constraints, given the endowment $K(0) = K_0$ and $A(0) = A_0$. In (2.8), ρ is the utility discount rate and the upper bound on s corresponds to investing all income in R&D and eating-up capital to finance consumption.

3 A competitive rationale

It might be objected that the optimal policy corresponding to (2.8) does not describe competitive behavior because it maximizes aggregate welfare ignoring the public good nature of knowledge. We present, therefore, a regulated competitive economy with a public good (knowledge) that gives rise to (2.8).

The stylized economy consists of an R&D sector, an output producing sector, identical households and a regulator. The R&D firms that transform the seed ideas into useful knowledge are financed by taxing household income at the rate $s(t)$.[2] Households own capital and labor, and derive income by renting these production factors to output producing firms.

Output producing firms operate in a competitive environment and seek to maximize instantaneous profit. At each point of time firm i rents capital K_i and hires labor L_i to produce a composite good according to the linearly homogenous production function $Y_i = F(K_i, AL_i) = AL_i f(k/A)$, taking as given the labor wage w, the capital rental rate r and the state of knowledge A, where $k = K_i/L_i$ and $f(x) = F(x, 1)$.[3] The capital demand condition is

$$f'(k/A) = r \tag{3.1}$$

and the labor market clearing condition requires that per worker profit vanishes, i.e.,

$$Af(k/A) - rk - w = 0. \tag{3.2}$$

From (3.2) and (2.6), the household income $y(t)$ is given by

$$y(t) = r(t)k(t) + w(t) = A(t)f(k(t)/A(t)) = Y(t)/L. \tag{3.3}$$

[2] This is evidently an abstraction, emphasizing the public good nature of knowledge and ignoring issues such as intellectual property rights and patent race.

[3] Firms using the same technology and facing the same market conditions employ factors at the same capital/labor ratio k hence the subscript i can be dropped.

Given that a fraction $s(t)$ of their income is collected as taxes to finance R&D, households allocate the remaining income $(1 - s(t))y(t)$ between consumption $c(t)$ and saving $\dot{k}(t)$,

$$\dot{k}(t) = (1 - s(t))y(t) - c(t), \tag{3.4}$$

and enjoy the instantaneous utility $u(c(t))$. The present value of a utility stream over the indefinite horizon is

$$\int_0^\infty u(c(t))e^{-\rho t}dt. \tag{3.5}$$

The household seeks the feasible consumption plan $c(t)$ that maximizes (3.5) subject to (3.4), given the initial capital k_0. In solving this problem, the household takes the tax policy $s(t)$ and the ensuing knowledge process $A(t)$ (treated as a public good and evolving according to (2.3)) as exogenous functions of time.

The regulator's role is to determine the tax policy $s(t)$. Multiplying (3.4) and (3.5) by L gives (2.7) and the objective of (2.8). Thus, the optimal tax policy, which fully accounts for the public good nature of knowledge, is determined by the social allocation problem (2.8).

4 Growth patterns

We characterize here the patterns of growth generated by (2.8), expressed more conveniently in terms of the per-capita stock $k = K/L$ and the per-capita unit cost of knowledge production

$$\varphi(A) = \Phi(A)/L.$$

The analysis is carried out in the (A, k) state space by means of two characteristic curves that divide the space into distinct regions and impose restrictions on the

R&D behavior in each of these regions. The salient features of the solution are presented here while the detailed derivations are relegated to the appendix.

The first curve, denoted $\tilde{k}(A)$, consists of the locus of (A, k) states satisfying

$$F_k(k, A) = F_A(k, A)/\varphi(A). \tag{4.1}$$

Equation (4.1) can be interpreted as a no-arbitrage condition, implying that along $\tilde{k}(A)$ the marginal productivity of capital investment equals that of knowledge investment. Recalling that $F(k, A) = Af(k/A)$, this condition reduces to $z(k/A) = \varphi(A)$, where the function $z(x) = f(x)/f'(x) - x$ is increasing with $z(0) = 0$. Solving for k, we obtain

$$\tilde{k}(A) = z^{-1}(\varphi(A))A. \tag{4.2}$$

As shown in the appendix, the curve $\tilde{k}(A)$ traces the unique locus in the (A, k) space along which both knowledge and capital grow simultaneously. The tendency to equate the marginal productivity of knowledge with that of capital stems from the observation that both factors compete for shares of the same income source. In the (A, k) region where capital is more productive at the margin, giving up some capital to finance knowledge accumulation implies an income loss exceeding the gain generated by the additional knowledge, hence R&D efforts are not warranted. Along $\tilde{k}(A)$ capital and knowledge are equally productive at the margin and the household is indifferent between investing in one or the other.

The second characteristic curve, denoted $\hat{k}(A)$, consists of the locus of (A, k) states where the capital rental rate $r = f'(k/A)$ (see 3.1) equals the discount rate ρ

$$f'(k/A) = \rho. \tag{4.3}$$

Since $f'(k/A) = F_k(k, A)$ is decreasing, (4.3) can be solved to yield

$$\hat{k}(A) = f'^{-1}(\rho)A \equiv \hat{\eta}A \tag{4.4}$$

and $\hat{k}(A)$ is the straight line emanating from the origin with the slope $\hat{\eta} = f'^{-1}(\rho)$.

The significance of the characteristic curves $\tilde{k}(A)$ and $\hat{k}(A)$ is stated in:

Proposition 1. *Economies that sustain long run growth first reach $\tilde{k}(A)$ at a most rapid R&D rate, setting $s(t) = 1$ while above $\tilde{k}(A)$ or $s(t) = 0$ while below it,[4] and evolve along $\tilde{k}(A)$ thereafter. Economies that fail to grow in the long run eventually reach a steady state on $\hat{k}(A)$.*

In view of the proposition, we refer to $\tilde{k}(A)$ and $\hat{k}(A)$ as the *turnpike* and the *stagnation line*, respectively. Economies that sustain growth in the long run are called growing, while those that converge to a steady state (on the stagnation line) are called stagnating. We turn now to specify the conditions for long run growth.

Let

$$\tilde{k}_\infty(A) = z^{-1}\left(\frac{1}{L\pi'(0)}\right) A \equiv \tilde{\eta} A \tag{4.5}$$

be the straight line emanating from the origin with the slope $\tilde{\eta} = z^{-1}\left(1/(L\pi'(0))\right)$. In view of (2.5) and (4.2), $\tilde{k}(A)$ approaches $\tilde{k}_\infty(A)$ as A increases and we refer to $\tilde{k}_\infty(A)$ as the asymptotic turnpike.[5] Since $\varphi(A)$ is non-increasing, the turnpike lies above the asymptotic line, $\tilde{k}(A) \geq \tilde{k}_\infty(A)$.

The slope difference $(\hat{\eta} - \tilde{\eta})$ determines the relative positions of the stagnation line and the asymptotic turnpike, and serves as the basis for the following growth condition:

Proposition 2. *Economies for which*

$$\hat{\eta} - \tilde{\eta} > 0 \tag{4.6}$$

holds have the potential to sustain long run growth. Economies for which the reverse condition holds eventually stagnate.

[4] An economy lies above (below) $\tilde{k}(A)$ if its capital exceeds (falls short of) $\tilde{k}(A)$ when its knowledge stock is A.

[5] $C_m(A) = O(A^m)$ and $C'_m(A) = O(A^{m-1})$, hence $\lim_{A\to\infty}[\tilde{k}(A) - \tilde{k}_\infty(A)] = 0$ for $m > 2$.

Having the potential to grow (i.e., satisfying condition (4.6)) does not ensure sustained growth, as realizing this potential requires sufficient resources (see Proposition 3 below). Condition (4.6) is readily interpreted. As stated in Proposition 1, a growing economy eventually evolves along the turnpike and its long-run marginal productivity of capital defines, according to (3.1), the long-run capital rental rate

$$r_\infty \equiv \lim_{A\to\infty} f'(\tilde{k}(A)/A) = \lim_{A\to\infty} f'(\tilde{k}_\infty(A)/A) = f'(\tilde{\eta}). \tag{4.7}$$

Thus, recalling that f' is decreasing, (4.6) is equivalent to

$$r_\infty = f'(\tilde{\eta}) > f'(\hat{\eta}) = f'(f'^{-1}(\rho)) = \rho, \tag{4.8}$$

i.e., the long-run equilibrium interest rate exceeds impatience – a familiar growth condition.

Consider now the intertemporal elasticity corresponding to $u(c)$ and assume that

$$\lim_{c\to\infty}\left\{\frac{-u''(c)c}{u'(c)}\right\} \equiv \sigma \geq 1. \tag{4.9}$$

The following characterization holds:

Proposition 3. *Suppose that the growth condition (4.6) is satisfied. To any initial knowledge stock A_0 there corresponds a threshold capital stock $k^{sk}(A_0) \geq 0$, such that when $k_0 \geq k^{sk}(A_0)$ the economy reaches in the long run a balanced growth path in which output, knowledge, capital and consumption all grow at the constant rate*

$$g_\infty = \frac{r_\infty - \rho}{\sigma} \tag{4.10}$$

and the constant income shares

$$s_\infty = \frac{g_\infty}{r_\infty}\left(\frac{1}{1+\tilde{\eta}L\pi'(0)}\right) \tag{4.11}$$

and

$$s_\infty^k = \frac{g_\infty}{r_\infty}\left(\frac{\tilde{\eta}L\pi'(0)}{1+\tilde{\eta}L\pi'(0)}\right) \tag{4.12}$$

are devoted to investments in knowledge and capital, respectively. If $k_0 < k^{sk}(A_0)$ *the economy eventually stagnates.*

A few observations are noteworthy. First, (4.10) agrees with the familiar Ramsey's rule (Ramsey 1928, equation 9). Second, (4.11) verifies that Weitzman's (1998) assumption (of a constant income share devoted to R&D) is consistent with optimizing behavior, albeit only in the long run and subject to the growth conditions. Third, a growing economy invests in the long run the share $s_\infty + s^k_\infty = g_\infty/r_\infty$ of its income and consumes the remaining share $1 - g_\infty/r_\infty$. Finally, the critical capital stock $k^{sk}(A_0)$ is akin to Skiba's (1978) threshold, hence the superscript 'sk'.

The long run exponential growth is evidently due to the asymptotic linear relation established between knowledge generation and output (cf. (2.3) and (2.5)), which gives rise to the linear asymptotic turnpike (4.5). As noted by Weitzman (1998), this asymptotic relation follows an intrinsic feature of the recombinant knowledge generating mechanism: the power of the combinatorial functions implies that H (the set of new seed ideas) grows faster than A. Indeed, (2.1) implies that when $\dot{A}/A$ is constant, $H = O(A^m)$. Because growth is driven by knowledge, output cannot outgrow knowledge and as the growth process proceeds, R&D expenditure per hybrid (sY/H) shrinks and so does the probability $\pi(sY/H)$ that the hybrid will yield a viable progeny idea. Eventually, the number of hybrids ceases to be a constraint and the limited resources available for R&D regulate the processing of new ideas such that knowledge, capital, output and consumption all grow at the same constant rate.

It is now possible to verify that the long run growth rate of (4.10) agrees with the general result of Weitzman (1998, equation 38): $g_\infty = F(s^k_\infty, s_\infty L\pi'(0))$. Recalling (cf. (4.7) and (4.5)) that $r_\infty = f'(\tilde{\eta})$ and $L\pi'(0) = 1/z(\tilde{\eta})$, we use (4.11), (4.12) and

$z(x) = f(x)/f'(x) - x$ to obtain $F(s_\infty^k, s_\infty L\pi'(0)) = s_\infty L\pi'(0) f(\tilde{\eta}) = \frac{g_\infty}{f'(\tilde{\eta})} \frac{f(\tilde{\eta})}{z(\tilde{\eta})+\tilde{\eta}} = g_\infty$. Here, however, the long run investment shares (s_∞ and s_∞^k defined in (4.11) and (4.12)) are endogenous variables that depend on the economy's underlying structure vis-à-vis its production technology F, R&D effectiveness π, and the preferences parameters ρ and σ.

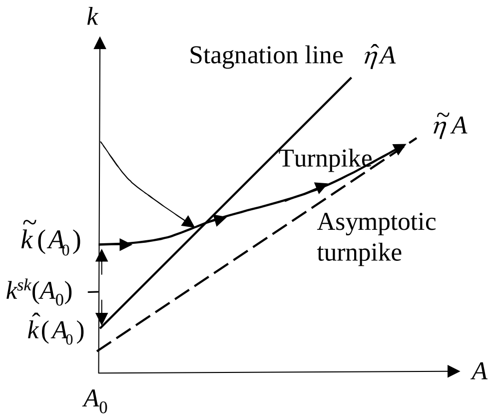

Figure 1: Knowledge-capital processes of economies that satisfy the growth condition (4.6) when $k^{sk}(A_0) < \tilde{k}(A_0)$. Arrows indicate direction of process evolution.

Figure 1 displays possible state-space trajectories of potentially growing economies (that satisfy the growth condition (4.6)) when $k^{sk}(A_0) < \tilde{k}(A_0)$. The threshold endowment $k^{sk}(A_0)$ separates economies into growing (above the threshold) and stagnating (below it). A growing economy with capital endowment between $k^{sk}(A_0)$ and $\tilde{k}(A_0)$ will initially avoid R&D to build up capital. As soon as its capital reaches $\tilde{k}(A_0)$, the economy tunes its consumption/saving rates so as to evolve along the turnpike (see the appendix). If capital endowment exceeds $\tilde{k}(A_0)$, R&D is initially financed at the maximal rate ($s = 1$) until the turnpike is reached, at which time the turnpike policy is adopted. As knowledge increases, the turnpike turns into a straight line (the asymptotic turnpike) and the economy approaches the balanced growth specified in Proposition 3. For a poorly endowed economy, with $k_0 < k^{sk}(A_0)$, R&D is never warranted and the economy eventually stagnates at $\hat{k}(A_0)$.

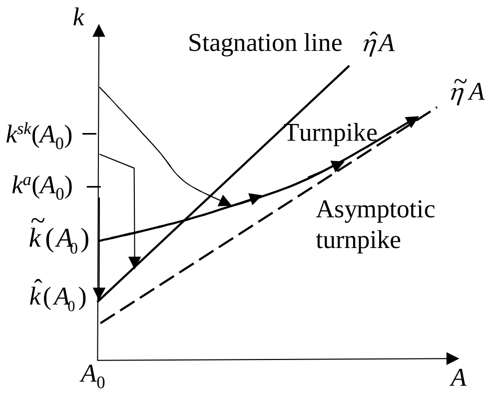

Figure 2: Knowledge-capital processes of economies that satisfy the growth condition (4.6) when $\tilde{k}(A_0) < k^{sk}(A_0)$. Arrows indicate direction of process evolution.

State-space trajectories of potentially growing economies when $k^{sk}(A_0) > \tilde{k}(A_0)$ are depicted in Figure 2. Here it is possible that an economy with capital endowment below $k^{sk}(A_0)$ (but above another threshold level $k^a(A_0)$) will temporarily invest in R&D at the maximal rate ($s = 1$). After a while, as capital decreases, R&D ceases to be attractive and is terminated abruptly. From that time onward the economy converges gradually, according to the one dimensional version of (2.8) with $s = 0$ and fixed A, to a point on the stagnation line below.

State-space trajectories of stagnating economies (that violate the growth condition (4.6)) are depicted in Figure 3. Except for capital endowments exceeding the threshold $k^1(A_0)$, R&D is not warranted and the economy approaches stagnation at $(A_0, \hat{k}(A_0))$. Capital endowments above $k^1(A_0)$ justify maximal R&D investment early on. At some point above the turnpike, however, knowledge accumulation is abruptly terminated, and the policy of decreasing capital brings the process gradually towards a steady state on the stagnation line below.

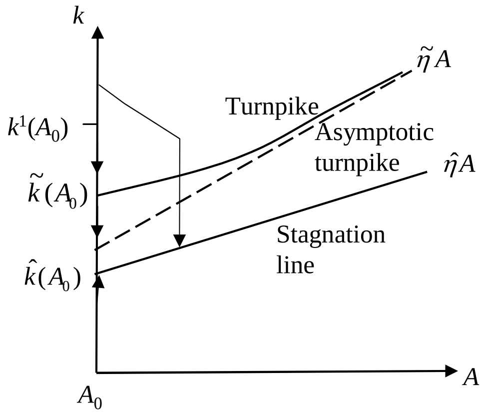

Figure 3: Knowledge-capital processes of economies that violate the growth condition (4.6). Arrows indicate direction of process evolution.

5 Example

We illustrate the above results for an economy with a Cobb-Douglas production technology $F(K, A) = \theta K^{\alpha}(AL)^{1-\alpha}$, $0 < \alpha < 1$. The turnpike specializes to

$$\tilde{k}(A) = \frac{\alpha}{1-\alpha}\varphi(A)A,$$

approaching the asymptotic line

$$\tilde{k}_{\infty}(A) = \frac{\alpha}{(1-\alpha)L\pi'(0)}A \equiv \tilde{\eta}A.$$

The stagnation line (4.4) reduces to

$$\hat{k}(A) = (\theta\alpha/\rho)^{\frac{1}{1-\alpha}}\, A \equiv \hat{\eta}A.$$

The large-A capital rental rate along the turnpike (cf. (4.7)) is

$$r_\infty = f'(\tilde{\eta}) = \theta\alpha \left(\frac{1-\alpha}{\alpha} L\pi'(0) \right)^{1-\alpha}$$

and the growth condition (4.6) (or (4.8)) becomes

$$\rho < \theta\alpha \left(\frac{1-\alpha}{\alpha} L\pi'(0) \right)^{1-\alpha} .$$

Endowed with sufficient capital, an economy satisfying the growth condition will reach the turnpike at a most-rapid-R&D rate and evolve along it thereafter. In the long-run, output, capital, knowledge and consumption all grow exponentially at the rate $g_\infty = (r_\infty - \rho)/\sigma$ by devoting to R&D and saving the constant income fractions $s_\infty = (1-\alpha)g_\infty/r_\infty$ and $s^k_\infty = \alpha g_\infty/r_\infty$, respectively. For a Cobb-Douglas economy the long-run income shares allocated to R&D and saving turn out to be proportional to the knowledge and capital shares in the production function.

6 Concluding comments

We extend Weitzman's (1998) recombinant framework to include endogenous R&D and saving decisions. Conditions under which long run growth is sustained are derived. Growing economies admit a turnpike behavior, approaching in the long run a balanced growth path under which the shares of income allocated to finance R&D, saving and consumption are constant. Weitzman's (1998) assumption of a constant R&D share is thus consistent with optimizing behavior in the long run. During earlier stages, however, R&D spending varies significantly, and can take the extreme options of no R&D or maximal R&D.

The necessary growth condition is formulated in terms of the slopes of two lines defined in the knowledge-capital state space, relating the economy's underlying structure to its long run growth potential. The stagnation line marks the eventual states of economies that fail to grow while the turnpike traces the growth trajectory.

Knowledge, represented by the stock of useful ideas, plays a dual role, augmenting labor productivity, on the one hand, and affecting the expected cost of producing a useful idea, on the other. As knowledge increases, the latter approaches a constant value, hence in the long run growth is driven solely by the labor augmenting role of knowledge. This state of affairs is manifest by the turnpike approaching a straight line (the asymptotic turnpike) at large knowledge levels and gives rise to the balanced, long run growth.

Growth failures occur either because the economy lacks the potential to sustain long run growth (i.e., violates the growth condition), or when it possesses this potential but lacks sufficient resources. The former situation may be due to poor social capital such as corruption, excessive bureaucracy or insufficient enforcement of property rights (Hall and Jones 1999) as well as to inappropriate capacity to exploit R&D effectively. External capital infusion can embark a poor, potentially-growing economy on a path of sustained growth but economies that violate the growth condition require structural changes in order to escape stagnation (Easterly 2003).

The method of analysis, based on the characteristic curves, is quite general and has been applied to other endogenous growth models (Tsur and Zemel 2004, 2005). An interesting extension of the present work might account for a competitive R&D sector, so that knowledge accumulation does not depend entirely on public spending and regulation policies.

Appendix

A Proofs of Propositions 1 and 2

Propositions 1 and 2 are based on a series of properties that restrict the behavior of the optimal (A, k) process in terms of its position relative to the characteristic curves. These properties are derived below.

Suppressing the time argument for brevity, the current-value Hamiltonian corresponding to the per-capita version of (2.8) is written as

$$H = u(c) + \lambda[(1-s)F(k,A) - c] + \gamma s F(k,A)/\varphi(A) \tag{A.1}$$

where λ and γ are the current-value costate variables associated with k and A, respectively. Necessary conditions for optimum include

$$u'(c) = \lambda, \tag{A.2}$$

$$s = \begin{cases} 1 & \text{if } \gamma/\varphi(A) > \lambda \\ 0 & \text{if } \gamma/\varphi(A) < \lambda \\ \tilde{s} & \text{if } \gamma/\varphi(A) = \lambda \end{cases} \tag{A.3}$$

($\tilde{s}$ is the singular R&D share defined in (A.8) below),

$$\dot{\lambda} - \rho\lambda = -F_k(k,A)\left[\lambda + s\left(\frac{\gamma}{\varphi(A)} - \lambda\right)\right], \tag{A.4}$$

$$\dot{\gamma} - \rho\gamma = -F_A(k,A)\left[\lambda + s\left(\frac{\gamma}{\varphi(A)} - \lambda\right)\right] + \frac{s\gamma F(k,A)\varphi'(A)}{\varphi(A)^2} \tag{A.5}$$

and the transversality conditions

$$(i)\ \lim_{t\to\infty} k\lambda e^{-\rho t} = 0, \quad (ii)\ \lim_{t\to\infty} \gamma e^{-\rho t} = 0. \tag{A.6}$$

Condition (A.3) identifies three possible R&D regimes, namely maximal R&D ($s = 1$), no R&D ($s = 0$) and singular R&D ($s = \tilde{s}$). The optimal policy selects among these three regimes at different phases of the planning horizon. Implementing the singular $\tilde{s}$ policy during a finite time interval is optimal only if the singular condition $\lambda = \gamma/\varphi(A)$ holds during this interval. Taking the time derivative and using (2.3) and (2.6), we find

$$\dot{\lambda} = \dot{\gamma}/\varphi(A) - \gamma s F(k, A)\varphi'(A)/\varphi(A)^3. \tag{A.7}$$

Using (A.4), (A.5) and the singular condition $\lambda = \gamma/\varphi(A)$, we write (A.7) as $F_k(k, A) = F_A(k, A)/\varphi(A)$, which is recognized as condition (4.1) defining the turnpike $\tilde{k}(A)$. Thus, the singular $\tilde{s}$ policy can proceed only along the turnpike and is given, in view of (2.3) and (2.7) by

$$\tilde{s} = [1 - c/y]\varphi(A)/[\tilde{k}'(A) + \varphi(A)]. \tag{A.8}$$

In a steady state, $\dot{A} = 0$ and (2.3) imply $s = 0$, which, together with $\dot{\lambda} = 0$ and (A.4), gives rise to $F_k = \rho$, in agreement with condition (4.3) that defines the stagnation line $\hat{k}(A)$. Thus, if the optimal policy ever approaches a steady state, this state must fall on the stagnation line. We state these observations for future reference as

Property 1. *(a) The singular policy $\tilde{s}$ can proceed only along the turnpike. (b) An optimal steady state falls on the stagnation line.*

Since $F_{kk} < 0$, the condition $\rho < F_k$ holds below $\hat{k}(A)$ (where $k < \hat{k}(A)$). Thus, (A.3) and (A.4) imply that $\dot{\lambda} < 0$ below $\hat{k}(A)$. This, together with (A.2) and $u''(c) < 0$, implies that $\dot{c} > 0$ below the stagnation line. The reverse relations ($\dot{\lambda} > 0$ and $\dot{c} < 0$) hold above $\hat{k}(A)$ if $s < 1$. We summarize these results in

Property 2. *The optimal consumption process increases in time below the stagnation line in all R&D regimes and decreases in time above this line under the no-R&D and singular regimes.*

Define

$$\Lambda(k, A) = F_k(k, A) - F_A(k, A)/\varphi(A). \tag{A.9}$$

According to (4.1), the turnpike is defined by $\Lambda(\tilde{k}(A), A) = 0$. Since $F_{kk} < 0$ and $F_{kA} > 0$, we find that

Property 3. $\Lambda(k, A) > 0$ *below the turnpike and* $\Lambda(k, A) < 0$ *above it.*

According to (A.3), the R&D policy is determined by the sign of $\xi = \gamma/\varphi(A) - \lambda$. Using (A.4), (A.5) and (A.9), we find $\dot{\xi} = \rho\xi + \Lambda(\lambda + s\xi)$, which is integrated to yield

$$\xi(t)e^{-\rho t} = \xi(t_0)e^{-\rho t_0} + \int_{t_0}^{t} \Lambda(k(\tau), A(\tau))[\lambda(\tau) + s(\tau)\xi(\tau)]e^{-\rho\tau}d\tau \tag{A.10}$$

for any arbitrarily chosen initial time t_0.

Since the shadow price λ is positive and $s\xi \geq 0$, (see A.3), the sign of the integrand is determined by Λ. Consider now the possibility that $\xi(t_0) > 0$ while the process evolves below the turnpike (where $\Lambda(k, A) > 0$). So long as the turnpike is not crossed, we see from (A.10) that

$$\xi(t)e^{-\rho t} > \xi(t_0)e^{-\rho t_0} > 0 \tag{A.11}$$

hence the R&D fraction remains fixed at the maximum value $s = 1$. This capital-decreasing regime, however, cannot cross the turnpike from below and will continue permanently, which implies that (A.11) violates the transversality conditions (A.6). It follows that the maximal R&D regime cannot be optimal below the turnpike. Similar considerations rule out the possibility that the no R&D regime holds indefinitely above the turnpike. Thus,

Property 4. *(a) Maximal R&D* ($s = 1$) *can be optimal only above the turnpike. (b) A steady state (with* $s = 0$*) cannot fall above the turnpike.*

In fact, maximal R&D can hold only during a finite period, otherwise the (A, k) process would cross the turnpike and violate Property 4a. After some time, this regime must either be replaced by a no-R&D policy above the turnpike or reach the turnpike (with $\xi = 0$) and switch to the singular policy.

Without R&D the capital process is monotonic in time because knowledge remains constant and the problem is essentially one-dimensional. Above the turnpike, the no-R&D policy involves decreasing capital (by consuming in excess of production) until the turnpike is reached (because by increasing k this regime would remain indefinitely above the turnpike, violating (A.6)). Now, ξ must be negative when the turnpike is reached from above under this policy. Since no other regime holds below the turnpike, this k-decreasing, constant-A plan must continue and converge to a steady state on the stagnation line segment below the turnpike.

Initiated below the turnpike, a no-R&D process cannot cross it. Neither can it switch to another regime below the turnpike. The only two possibilities left are to converge to a steady state below the turnpike or to reach the turnpike (with $\xi = 0$) and switch to the turnpike policy. We summarize these considerations in

Property 5. *(a) A maximal R&D policy initiated above the turnpike can proceed only during a finite time interval, following which it is replaced by either a no-R&D policy (above the turnpike) or a singular policy (on the turnpike). (b) A no-R&D policy initiated above the turnpike continues permanently and the ensuing* (A, k) *process converges to a steady state on the stagnation line segment below the turnpike. (c) A no-R&D policy initiated below the turnpike either drives the (A,k) process to a steady state on the stagnation line (below the turnpike) or is replaced by the singular policy upon reaching the turnpike.*

Once the singular policy has been initiated along the turnpike (with $\xi = 0$ and

$\dot{\xi} = 0$) we find, using (A.10), that the (A, k) process cannot leave the turnpike without violating Properties 4 or 5 (in other words, the singular policy is trapping). In view of Property 1, this policy can either converge to a steady state at the intersection point $(\hat{A}, \hat{k})$ of $\tilde{k}(A)$ and $\hat{k}(A)$ (if such a point exists) or grow indefinitely along the turnpike. The first possibility can be ruled out. Consider a singular policy confined indefinitely to the turnpike segment above the stagnation line. According to Property 2, this involves a decreasing consumption process. However, the policy of staying at the initial state (diverting to consumption the resources allocated by the singular policy to increase the capital and knowledge stocks) is feasible and yields a higher utility. Therefore, the singular policy that drives the (A, K) process permanently above the stagnation line is not optimal. The geometry of the characteristic curves of the present model is such that the turnpike can cross the stagnation line only from above (see the discussion following (4.5)). It follows that a singular process converging to $(\hat{A}, \hat{k})$ always lies above $\hat{k}(A)$ and cannot be optimal. These considerations imply

Property 6. *A singular policy cannot be confined to a turnpike segment above the stagnation line and must proceed indefinitely along the turnpike.*

When the growth condition (4.6) is violated, the stagnation line lies below the turnpike at all knowledge levels. From Property 6 we conclude that

Property 7. *(a) When the growth condition (4.6) is violated, the singular policy cannot be optimal. (b) When the growth condition (4.6) is satisfied, the singular policy implies sustained growth.*

We can now use these properties to derive Propositions 1 and 2. Observe that capital and knowledge can grow simultaneously only under the singular regime.

Proof of Proposition 1: Follows from Properties 1, 4a, 5 and 6.

Proof of Proposition 2: When condition (4.6) is violated, Property 7a forbids the singular policy. According to (A.3) the economy can either engage in maximal R&D ($s = 1$) or no R&D ($s = 0$). Since the first possibility can only be implemented temporarily (Property 5a) the no-R&D policy must eventually be implemented and, according to Property 5b-c, the economy approaches a steady state on the stagnation line. When the growth condition (4.6) holds, the singular policy cannot be ruled out and the economy bears the potential to sustain long run growth.

B Proof of Proposition 3

The proof is based on the following derivation of the turnpike investment-consumption decisions. Along the turnpike, capital changes with knowledge according to $\tilde{k}(A)$ of (4.2). Letting a '$\tilde{\ }$' symbol over any variable represent its turnpike trajectory, (2.3) is rewritten as

$$\dot{A} = \tilde{s}\tilde{y}(A)/\varphi(A), \tag{B.1}$$

where

$$\tilde{y}(A) = F(\tilde{k}(A), A) = Af(\tilde{k}(A)/A) \tag{B.2}$$

and (2.7) takes the form

$$\dot{\tilde{k}} = (1 - \tilde{s})\tilde{y}(A) - \tilde{c} = \tilde{k}'(A)\dot{A} = \tilde{k}'(A)\tilde{s}\tilde{y}(A)/\varphi(A). \tag{B.3}$$

The turnpike consumption $\tilde{c}$ is thus given by

$$\tilde{c} = \tilde{y}(A)[1 - \tilde{s}(\varphi(A) + \tilde{k}'(A))/\varphi(A)] \tag{B.4}$$

which implies

$$d\tilde{c}/d\tilde{s} = -\tilde{y}(A)(\varphi(A) + \tilde{k}'(A))/\varphi(A). \tag{B.5}$$

From (B.2) we find

$$\tilde{y}'(A) = F_A(\tilde{k}(A), A) + F_k(\tilde{k}(A), A)\tilde{k}'(A) = \tilde{r}(A)(\varphi(A) + \tilde{k}'(A)), \tag{B.6}$$

where it is recalled that $F_A = F_k\varphi(A)$ holds along the turnpike and

$$\tilde{r}(A) = F_k(\tilde{k}(A), A). \tag{B.7}$$

The turnpike policy is the outcome of

$$\tilde{V}(A_0) = \max_{\{\tilde{s}(t)\}} \int_0^\infty u(\tilde{c}(t))e^{-\rho t}dt \tag{B.8}$$

subject to (B.1) and (B.4), $0 \leq \tilde{s} \leq \varphi(A)/(\varphi(A) + \tilde{k}'(A))$ and other feasibility (e.g., non-negativity) constraints, given the endowment $A(0) = A_0$. Let $\tilde{m}$ be the current-value shadow price of knowledge along the turnpike. The current-value Hamiltonian is

$$\tilde{H} = u(\tilde{c}) + \tilde{m}\tilde{s}\tilde{y}(A)/\varphi(A) \tag{B.9}$$

and necessary conditions for internal optimum include (see B.5)

$$u'(\tilde{c}) = \frac{\tilde{m}}{\varphi(A) + \tilde{k}'(A)}, \tag{B.10}$$

$\dot{\tilde{m}} - \rho\tilde{m} = -\partial H/\partial A$, which reduces, noting (B.4), (B.6) and (B.10), to

$$\frac{\dot{\tilde{m}}}{\tilde{m}} = \rho - \tilde{r}(A) + \frac{\tilde{s}\tilde{y}(A)(\varphi'(A) + \tilde{k}''(A))}{(\varphi(A) + \tilde{k}'(A))\varphi(A)} \tag{B.11}$$

and the transversality condition (Michel 1982)

$$\lim_{t\to\infty} \tilde{H}(t)e^{-\rho t} = 0. \tag{B.12}$$

Taking the time derivative of (B.10) gives

$$u''(\tilde{c})\dot{\tilde{c}}(\varphi(A) + \tilde{k}'(A)) + u'(\tilde{c})(\varphi'(A) + \tilde{k}''(A))\tilde{s}\tilde{y}(A)/\varphi(A) = \dot{\tilde{m}},$$

which reduces, using (B.10) and (B.11), to $\dot{\tilde{c}}u''(\tilde{c})/u'(\tilde{c}) = \rho - \tilde{r}$ or

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{\tilde{r} - \rho}{\tilde{\sigma}} \equiv \tilde{g}, \tag{B.13}$$

where $\tilde{\sigma}(\tilde{c}) = -\tilde{c}u''(\tilde{c})/u'(\tilde{c})$ is the intertemporal elasticity of substitution.

Define

$$q(A) = \tilde{s}(\varphi(A) + \tilde{k}'(A))/\varphi(A) \tag{B.14}$$

and rewrite (B.4) as

$$\tilde{c} = \tilde{y}(A)(1 - q(A)), \tag{B.15}$$

so that q is the income share invested in capital and knowledge along the turnpike. Taking the time derivative of (B.15), using (B.1) and (B.6), we find

$$\dot{\tilde{c}} = \tilde{y}\tilde{r}q(1 - q) - \tilde{y}\dot{q}. \tag{B.16}$$

Dividing (B.16) by (B.15) gives

$$\dot{q}/(1 - q) - \tilde{r}q + \tilde{g} = 0. \tag{B.17}$$

Using (B.13) to rewrite (B.17) as

$$\frac{-dlog(1 - q)}{dt} + \frac{dlog(\tilde{c})}{dt} = \tilde{r}q \tag{B.18}$$

and integrating, recalling that $\tilde{y} = \tilde{c}/(1 - q)$ (see B.15), we obtain

$$\tilde{y}(A(t)) = \tilde{y}(A(t_0))exp\left(\int_{t_0}^{t} \tilde{r}(\tau)q(\tau)d\tau\right) \tag{B.19}$$

for any $t \geq t_0 \geq 0$. Thus, output grows exponentially when the rate $\tilde{r}$ and the income share q approach their constant asymptotic values.

To verify that the share q is asymptotically constant, we recall that according to (4.7) and (4.9), the large A and c limits of $\tilde{r}$ and $\tilde{\sigma}$ are given by r_∞ and σ, respectively. Noting (B.13), we find that as the economy grows $\tilde{g}$ approaches

$$g_\infty = (r_\infty - \rho)/\sigma, \tag{B.20}$$

in agreement with (4.10). We can now return to (B.17) with the constant parameters $\tilde{r} = r_\infty$ and $\tilde{g} = g_\infty$ and obtain the general solution along the asymptotic turnpike

$$q(t) = \frac{1 + \psi exp[(r_\infty - g_\infty)t]}{r_\infty / g_\infty + \psi exp[(r_\infty - g_\infty)t]} \ ,$$

where ψ is an integration constant. Excluding solutions that diverge at a finite time and recalling that $r_\infty > g_\infty$, we find that any $\psi \neq 0$ gives rise to $q \to 1$, implying that the long run consumption share shrinks to zero. This solution, however, cannot be optimal because (B.19) implies that $\tilde{y}$ grows at the rate r_∞ while $\tilde{m}$ shrinks at the rate $\rho - r_\infty$, hence the second term of the Hamiltonian (B.9) grows at the rate ρ, violating (B.12). Thus, $\psi = 0$, $q = g_\infty / r_\infty$ and (B.19) implies that in the long run $\tilde{y}$ increases exponentially at the rate g_∞. Taking the large A limit of (B.14) we find

$$s_\infty = \frac{g_\infty}{r_\infty} \left(\frac{1}{1 + \tilde{\eta} L \pi'(0)} \right) \tag{B.21}$$

and (B.3) gives

$$s_\infty^k = \frac{g_\infty}{r_\infty} \left(\frac{\tilde{\eta} L \pi'(0)}{1 + \tilde{\eta} L \pi'(0)} \right), \tag{B.22}$$

verifying (4.11) and (4.12). With $\tilde{k}(A)$ approaching $\tilde{k}_\infty(A) = \tilde{\eta} A$, (B.2) implies that knowledge and capital are proportional to $\tilde{y}$ and grow at the same rate g_∞.

To understand the Skiba behavior when the characteristic curves cross, note, using Properties 1 and 4b, that there exists a minimal knowledge level $A_0 \leq A_m \leq \hat{A}$ such that when initiated from $(A_m, \tilde{k}(A_m))$ on the turnpike, the optimal process must follow the singular policy of unbounded growth. When $A_m > A_0$, the critical level $k^{sk}(A_0)$ is obtained by solving the dynamic equations for A, k and λ backward in time with $s = 1$, using the initial values $A = A_m$ and $k = \tilde{k}(A_m)$ and the initial value of λ determined by the condition that the no R&D policy ($s = 0$) that drives the system from $(A_m, \tilde{k}(A_m))$ to a steady state $(A_m, \hat{k}(A_m))$ on the stagnation line is also optimal. The Skiba point $k^{sk}(A_0)$ corresponds to the capital stock at the (reversed) time when the knowledge endowment A_0 is reached, and lies above $\tilde{k}(A_0)$, as shown in Figure 2. When $A_m = A_0$, the Skiba point lies between $\tilde{k}(A_0)$ and $\hat{k}(A_0)$, as shown in Figure 1. If the endowment $\hat{k}(A_0)$ justifies increasing capital until the turnpike is reached, followed by singular growth thereafter, set $k^{sk}(A_0) = 0$.

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