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Dynamic-spatial management of coastal aquifers

by

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Dynamic-spatial management of coastal aquifers

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Abstract

We analyze the management of a coastal aquifer under seawater intrusion using distributed control methods. The aquifer's state is taken as the water head elevation, which varies with time and in space since extraction, natural recharge and lateral water flows vary with time and in space. The water head, in turn, induces a temporal-spatial seawater intrusion process, which changes the volume of fresh water in the aquifer. Under reasonable conditions we show that the optimal state converges to a steady state process that is constant in time. We characterize the optimal steady state process in terms of a standard control problem (in space) and offer a tractable algorithm to solve for it.

Keywords: distributed control; groundwater; optimal exploitation; seawater intrusion;

JEL Classification: C61, C62, Q25

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1 Introduction

Most economic models of natural resource exploitation are dynamic in nature, accounting for the evolution of the resource stock over time as a result of human use and natural regeneration processes. Often the resource state evolves in time but varies across space as well. Examples include: (i) coastal aquifers under seawater intrusion (SWI) that depends on the aquifer's water head, which varies across space as a result of the spatial variability of pumping policies; (ii) the density of fish population that evolves over time due to harvesting and net regeneration and changes from location to location as a result of migration due to competition for food; (iii) the age-density distribution of a forest; and (iv) the concentration of air, soil, or water pollution. The resource state in such cases varies with time and with the spatial location. Adding the spatial dimension turns the equation describing the motion of the resource state from an ordinary differential equation to a partial differential equation and a management policy entails exploitation rates that are time and space dependent. Consequently, the resource management problem changes from that of a standard optimal control to distributed control (see [4]). In this paper we apply distributed control methods to study the management of coastal aquifers under SWI.

Optimal groundwater management is associated with a wide range of considerations, such as temporal and spatial hydrological processes, external impacts of surface activities on water quality, stock-dependent extraction costs, uncertainty with respect to natural recharge and flows between adjacent water bodies, and the effect of market structure in the water economy. There is an extensive economic literature on the subject. Early studies [3, 2] investigated optimal groundwater withdrawals from a single-cell aquifer. Welfare implications of extraction under common property and groundwater property rights were studied by [8]. Knapp and Feinerman [16] extended the spatial framework to a multi-cell aquifer, analyzing optimal steady-state solutions. Other extensions comprise game theoretic strategic behavior in extraction [21, 22] and conjunctive use of surface and groundwater [27, 17]. Many studies examine dynamic implications of agricultural activities on groundwater quality. Knapp et al. [18] and Shah et al. [25] viewed unconfined aquifers as an exhaustible resource for storage of saline drainage, where subsurface drainage-system constitutes a backstop technology. Other studies explicitly introduce groundwater quality as a state variable [11, 26, 6, 15]. Kim et al. [14] consider the time lag in the effect of deep percolation on groundwater quality, while [13] incorporates the spread of impact over time due to the spatial nature of groundwater flows toward aquifer outlets.

We study optimal management of coastal aquifers where the groundwater stock determines the extent of SWI, hence also the effective aquifer capacity containing fresh water. Cummings [5] was among the first to develop a generalized model of groundwater extraction under the presence of SWI. He considered water transfers to intruded areas as a mitigation strategy. Tsur and Zemel [28, 29] analyzed SWI as an adverse abrupt event, occurring at an uncertain date due to conditions that are not fully understood, where the exploitation policy affects the occurrence probability. They characterized optimal management under various types of events (reversible and irreversible) and uncertainty (endogenous and exogenous). Reinelt [24] considered the SWI as a deterministic and gradual process. He analyzed optimal extraction regimes from a confined coastal aquifer while incorporating both the temporal and spatial dimensions of the problem. Reinelt [24] utilized Darcy's law to express the dynamic-spatial management problem in terms of partial differential equations of motion. However, instead of characterizing the optimal policy analytically, he resorted to numerical methods based on discrete approximations.

The paucity of analytical dynamic-spatial models in economic literature is likely due to their complexity. There are but a few examples of such models in resource management contexts, including [23] on forest harvesting, [1] on river contamination by runoff, [32] on capital under environmental taxes, [9] on fertilizing-driven runoff, and [12] on cropping under the influence of pathogens. Drawing on recent results by Leizarowitz [19], we develop such a model for coastal aquifers.

The aquifer's state and its motion in time and space are characterized in Section 2. Section 3 formulates the dynamic-spatial management problem as a distributed control problem. In Section 4 we characterize the optimal extraction policy in time and space and show (under reasonable assumptions) that the system converges to a steady state in which the various processes do not change over time. In Section 5 we characterize the steady state policy in terms of a standard control problem in space. In section 6 we offer a computationally tractable procedure to solve the distributed control problem (the steady state policy and the transition to it). Section 7 concludes.

2 Dynamic-spatial formulation

We consider a homogeneous, unconfined coastal aquifer extending east of a north-south coastline. Let x measure the distance from the sea: $0 \leq x \leq L$, where $x = 0$ at the seashore and $x = L$ at the eastern end of the aquifer. Let $F(x)$ measure the north-south length of the aquifer at location x. Consider a strip of the aquifer at location x with sides $F(x)$ and Δx, denoted $D_x(\Delta x)$ (see Figure 1). Wells are distributed at density $\lambda(x)$ per unit area, thus $D_x(\Delta x)$ contains approximately $\lambda(x)F(x)\Delta x$ wells.[1]

Figure 1

Our model is time dependent and we denote by $g(x,t)F(x)\Delta x$ the rate of water extraction from $D_x(\Delta x)$ at time t (dimension m^3t^{-1}, m= meter). The natural replenishment (recharge) rate at $D_x(\Delta x)$ and time t due to rainfall, deep percolation, and subsurface flows between the aquifer and other groundwater bodies, denoted $r(h(x,t))$, is assumed to be dependent

[1] When the north-south length of the aquifer is large, the variability along this direction may be substantial, violating the homogeneity assumption. Often, as in the Israeli case, the aquifer can be divided into homogenous sections (or cells). Each section is then treated separately and later integrated into a model of the whole aquifer. Here we assume that the homogeneity assumptions holds.

on the average water-head level $h(x,t)$ in $D_x(\Delta x)$ above the sea level. Thus, $r(h(x,t))F(x)\Delta x$ is the average net external recharge at $D_x(\Delta x)$. Net external outflow from $D_x(\Delta x)$ during $[t, t+\Delta t]$ is thus

$$[g(x,t) - r(h(x,t))]F(x)\Delta x\Delta t.$$

Let $\mu(x,t)F(x)\Delta x$ represent the rate of net lateral flow at $D_x(\Delta x)$, i.e., net flow from its contiguous sections. Lateral flows are driven by changes in the water head $h(x,t)$ and are specified below. The water balance equation for $D_x(\Delta x)$ during $[t, t+\Delta t]$ is given by

$$[h(x,t+\Delta t) - h(x,t)]\phi(x)F(x)\Delta x = [r(h(x,t)) - g(x,t) + \mu(x,t)]F(x)\Delta x\Delta t, \tag{2.1}$$

where $\phi(x)$ is the aquifer's porosity parameter.

We now specify the lateral flow term $\mu(x,t)$. Let $Q(x,t)$ be the rate of water (m^3t^{-1}) going through the rectangle of sides $F(x)$ and $h(x,t)$ from one side ($x-$) to the other ($x+$). From Darcy's law

$$Q(x,t) = -\kappa\frac{\partial h(x,t)}{\partial x}h(x,t)F(x) \tag{2.2}$$

and

$$Q(x+\Delta x,t) = Q(x,t) + \frac{\partial}{\partial x}\left(-\kappa\frac{\partial h(x,t)}{\partial x}h(x,t)F(x)\right)\Delta x + o(\Delta x) \tag{2.3}$$

where κ is the aquifer's hydraulic conductivity. Thus, ignoring $o(\Delta x)$ terms,

$$\mu(x,t)F(x)\Delta x = Q(x,t) - Q(x+\Delta x,t) = \frac{\partial}{\partial x}\left(\kappa \frac{\partial h(x,t)}{\partial x} h(x,t)F(x)\right)\Delta x \tag{2.4}$$

or

$$\mu(x,t) = \frac{\kappa}{2}\left(\frac{\partial^2 h^2(x,t)}{\partial x^2} + \frac{\partial h^2(x,t)}{\partial x}\frac{F'(x)}{F(x)}\right). \tag{2.5}$$

When $F(x)$ varies "slowly" with x, such that $\frac{|F'(x)|}{F(x)}$ is negligibly small, $\mu(x,t)$ can be approximated by

$$\mu(x,t) = \frac{\kappa}{2}\frac{\partial^2 h^2(x,t)}{\partial x^2},$$

but we will not use this approximation in the present discussion.

Introducing the notation

$$H(x,t) = h^2(x,t), \tag{2.6}$$

we obtain, recalling (2.1) and (2.5),

$$\frac{\phi(x)}{h(x,t)}H_t(x,t) = \kappa H_{xx}(x,t) + \kappa\frac{F'(x)}{F(x)}H_x + 2\left(r(h(x,t)) - g(x,t)\right). \tag{2.7}$$

The boundary conditions associated with equation (2.7) include the initial water head levels

$$h(x,0) = h_0(x),\ 0 \le x \le L \tag{2.8}$$

and appropriate boundary conditions at $x = 0$ and $x = L$, such as

$$h(0,t) = 0,\ h_x(0,t) = b_0(t),\ h_x(L,t) = b_L(t),\ 0 \le t \le L. \tag{2.9}$$

We still have to combine this dynamics for h, expressed as a nonlinear parabolic equation, with the reward functional over a long time period. This will yield a distributed optimal control problem for the agriculture-economic system which we study. Once this is formulated precisely, we will indicate the method of solution that we propose.

A desirable feature of the distributed optimal control that we formulated is that when considered on long time intervals the corresponding optimal solutions will tend to some steady state equilibrium, which doesn't depend on the initial state. The state equation (2.7) is nonlinear, and in general establishing convergence to a steady state for nonlinear equations is a difficult task. Nevertheless, we managed to establish this under the assumption that the recharge function r is linear in H:

$$r(h) = a_0 - bh^2 = a_0 - bH, \ 0 \leq H \leq a_0/b. \tag{2.10}$$

The existence of a unique steady state equilibrium $z^\star$ is considered in section 4. Once this is established then the characterization of $z^\star$ follows from standard results in finite dimensional calculus of variations.

3 The management problem

The average water head (relative to sea level) at strip $D_x(\Delta x)$ is represented by the function $h(x,t)$. It induces induces a seawater intrusion function $f(h)$, such that at time t length $f(h(x,t))$ out of the whole length of $D_x(\Delta x)$ is saline, and the length $F(x) - f(x)$ is fresh.[2] The number of fresh water wells in location x is $[F(x) - f(h(x,t))]\lambda(x)\Delta x$ and, if water is extracted only from fresh water wells, then the extraction rate per well in location x with water-head level $h(x,t)$ is

$$e(h, g, x) = \frac{F(x)g(x,t)}{[F(x) - f(h(x,t))]\lambda(x)}. \tag{3.1}$$

It is expedient to define $\theta(x, h)$ as

$$\theta(x, h) = \frac{f(h)}{F(x)},$$

so that at time t and location x the length $\theta(x, h(x,t))F(x)$ is saline. Using this variable, the extraction rate per well at location x with water-head level $h(x,t)$ is

$$e(h, g, x) = \frac{g(x,t)}{[1 - \theta(x, h(x,t))]\lambda(x)}. \tag{3.2}$$

Suppressing the dependence of θ on the x variable we suppose that $\theta(h)$ is monotone decreasing, satisfies $0 \le \theta(h) \le 1$ and is such that

$$\theta(0) \approx 1 \text{ and } \theta(h) = 0 \text{ for } h > h_M$$

for some constant h_M.

We assume that the extraction cost per (operating) well depends both on the water head h and on the per-well extraction rate e, and denote it $c(h, e)$. The extraction cost in the rectangular strip of edges Δx and $[1 - \theta(h)]F(x)$

[2] The specification of the intrusion function $f(h)$ is based on the Ghyben-Herzberg principle.

is obtained from multiplying $c(h,e)$ by the number of operating wells $[1-\theta(h)]F(x)\lambda(x)\Delta x$. Aggregate extraction cost at time t is therefore

$$C(t) = \int_0^L c(h,e)[1-\theta(x,h)]F(x)\lambda(x)dx. \tag{3.3}$$

We assume that $c(h,0) = 0$ (extracting a zero rate doesn't inflict cost), $c(h,e)$ is increasing and strictly convex in e, and is decreasing in h. We shall use the following specifications:

$$c(h,e) = \tilde{c}(h)e^p \tag{3.4}$$

and

$$\tilde{c}(h) = \frac{a}{(\alpha h+\beta)^\mu} \tag{3.5}$$

for some positive constants a, α, β and μ, and $p > 1$. Using (3.2) and (3.4), the aggregate extraction cost at time t in (3.3) is

$$C(t) = \int_0^L \frac{\tilde{c}(h)}{(1-\theta)^{p-1}\lambda^{p-1}} g^p F(x)dx. \tag{3.6}$$

The function $\theta(h)$ decreases monotonically and vanishes at $h = h_M$, and we have $0 \leq \theta(h) \leq 1$ for every $0 \leq h \leq h_M$, and $h(0) \approx 1$. We choose for $\theta(h)$ the form

$$\theta(h) = 1 - \left(\frac{\alpha h+\beta}{\alpha h_M+\beta}\right)^\nu \tag{3.7}$$

where α and β are as in (3.5), and

$$0 < \nu < 1.$$

Choosing the coefficients α and β the same in (3.5) and (3.7) is very helpful in rendering our analysis and our computations tractable. We have, however, enough freedom provided by the other parameters a, ν and μ to match experimental data with the expressions (3.5) and (3.7).

Substituting these expressions in (3.6) we obtain

$$C(t) = \int_0^L \frac{A(x)}{(\alpha h + \beta)^{\mu + \nu(p-1)}} g^p F(x) dx \tag{3.8}$$

where

$$A(x) = \frac{a(\alpha \bar{h} + \beta)^{\nu(p-1)}}{\lambda(x)^{p-1}}. \tag{3.9}$$

The dependence of the integrand in (3.8) on the variables (h, g) is of the form

$$\Phi(h, g) = \frac{g^p}{(\alpha h + \beta)^\rho}, \tag{3.10}$$

where $\rho = \mu + \nu(p-1)$. We need the following simple result:

Lemma 3.1 *Let* $\Phi(\cdot, \cdot)$ *be the function in* (3.10) *with* $\rho, p > 0$, *and consider it on the domain* $h, g > 0$. *If*

$$\rho + 1 < p$$

then $\Phi(\cdot, \cdot)$ *is strictly convex. If* $\rho + 1 = p$ *then* Φ *is convex. Moreover, under the same conditions also the function*

$$\Psi(H, g) = \frac{g^p}{(\alpha \sqrt{H} + \beta)^\rho} \tag{3.11}$$

is convex in (H, g).

Proof We compute the second order derivatives of Φ:

$$\Phi_{gg} = \frac{p(p-1)g^{p-2}}{(\alpha h+\beta)^{\rho}},\ \Phi_{hh} = \frac{\alpha^2\rho(\rho+1)g^p}{(\alpha h+\beta)^{\rho+2}} \text{ and } \Phi_{gh} = -\frac{\alpha p\rho g^{p-1}}{(\alpha h+\beta)^{\rho+1}}.$$

We have then that

$$\Phi_{gg}\cdot\Phi_{hh}-(\Phi_{gh})^2 = \frac{\alpha^2 p(p-1)\rho(\rho+1)-(\alpha p\rho)^2}{(\alpha h+\beta)^{2\rho+2}}g^{2p-2} = \frac{\alpha^2\rho p(p-\rho-1)g^{2p-2}}{(\alpha h+\beta)^{2\rho+2}}$$

and the assertion concerning Φ follows.

Concerning Ψ we have the following expressions

$$\Psi_{gg} = \frac{p(p-1)g^{p-2}}{(\alpha\sqrt{H}+\beta)^{\rho}},\ \Psi_{H} = -\frac{\alpha\rho g^p}{2\sqrt{H}(\alpha\sqrt{H}+\beta)^{\rho+1}}$$

and

$$\Psi_{HH} = \frac{\alpha^2\rho(\rho+1)g^p}{4H(\alpha\sqrt{H}+\beta)^{\rho+2}} + \Delta \text{ and } \Psi_{gH} = -\frac{\alpha p\rho g^{p-1}}{2\sqrt{H}(\alpha\sqrt{H}+\beta)^{\rho+1}}.$$

where $\Delta > 0$. Using these expressions and the positivity of Δ we obtain

$$\Psi_{gg}\cdot\Psi_{HH}-(\Psi_{gH})^2 = \frac{\alpha^2\rho p(p-\rho-1)g^{2p-2}}{4H(\alpha\sqrt{H}+\beta)^{2\rho+2}} + \frac{p(p-1)g^{p-2}\Delta}{(\alpha\sqrt{H}+\beta)^{\rho}} > 0$$

since both terms are positive. The proof of the Lemma is complete.

Employing the lemma to the function forming the integrand in (3.8) we conclude that it is convex if $p \geq \mu+\nu(p-1)+1$, which holds when

$$p > \frac{\mu}{1-\nu}+1 \tag{3.12}$$

if $0<\nu<1$. We will assume henceforth that (3.12) holds.

We turn now to the benefit expression. The extracted water $g(x,t)$ generates the instantaneous benefit $U(g(x,t))$ at time t, where the benefit function $U(\cdot)$ is assumed to be increasing and strictly concave. An extraction policy

$$\Gamma = \{g(x,t) : 0 \le x \le L, 0 \le t < \infty\}$$

is feasible if it satisfies:

$$0 \le g(x,t) \le k(x),\ 0 \le h(x,t) \le h_M(x),\ 0 \le x \le L,\ t \ge 0, \tag{3.13}$$

for some function $k(x)$ that expresses the maximal extraction capacity, and a constraint function $h_M(x)$. We seek for a feasible policy that maximizes

$$\int_0^\infty \int_0^L U(g(x,t))F(x)dx - C(t)dt \tag{3.14}$$

subject to (2.7) and (3.13), for prescribed initial conditions (2.8) and boundary conditions (2.9).[3] The cost $C(t)$ in (3.14) is given by (3.8).

In our model the dynamics are described by equation (2.7), where the variable $h(x,t)$ is the state of the system at time t and location x, $g(x,t)$ is the distributed control variable, and these variables should satisfy (3.13). We will henceforth assume the following form for the utility function:

$$U(g) = bg^\gamma$$

[3]The discounted version of (3.14) is

$$\int_0^\infty e^{-\rho t} \left[\int_0^L U(g(x,t))F(x)dx - C(t) \right] dt$$

with ρ as the time rate of discount. The discounted problem turns out to be more technically involved and is left for future research.

for some constant $b > 0$, and for a constant $0 < \gamma < 1$. This yields the benefit contribution

$$B = b \int_0^T \int_0^L (g(x,t))^\gamma F(x) dx dt, \tag{3.15}$$

The instantaneous cost expression is given by (3.8), so that the total cost is

$$C = \int_0^T \int_0^L \frac{A(x)}{(\alpha h + \beta)^\rho} g^p F(x) dx dt, \tag{3.16}$$

In (3.16) the exponent in the denominator is

$$\rho = \mu + \nu(p-1),$$

which by (3.12) satisfies $\rho + 1 < p$. In view of Lemma 3.1 it follows that the integrand in (3.16) is a strictly convex functions of (h, g). The resulting payoffs is

$$R = \int_0^T \int_0^L \left\{ g^\gamma - \frac{A(x)}{(\alpha h + \beta)^\rho} g^p \right\} F(x) dx dt \tag{3.17}$$

We denote the integrand in the expressions above by

$$\mathcal{L}(h, g, x) = g^\gamma - \frac{A(x)}{(\alpha h + \beta)^\rho} g^p, \tag{3.18}$$

and note, recalling Lemma 3.1, that it is strictly concave in (h, g). The aquifer management problem is to find the feasible extraction policy

$$\Gamma = \{g(x,t),\ 0 \leq x \leq L,\ 0 \leq t \leq T\}$$

for some large but finite T that maximizes the payoff subject to (2.7)-(2.9).

4 Existence of and convergence to steady state

We consider the maximization of the undiscounted total reward $R(T, g)$ in (3.17) subject to (2.7), and take the limit of $R(T, g)$ as T increases to infinity. Since this limit diverges, we seek the policies g that maximize the long-run average rewards $\frac{1}{T}R(T, g)$. Actually we consider policies $g^\star$ that in addition to having maximal long-run average reward have the following property: For every feasible policy g there exists a constant M such that

$$R(T, g^\star) > R(T, g) - M$$

for every $T > 0$. Such policies are called *good policies* (see [19]). In particular, good policies have maximal long-run average reward.

In a steady state the various processes are independent of time. In this section we show that *good policies* corresponding to the aquifer management problem converges to a steady state. Our analysis relies on properties established in [19]. To keep our work self contained, we briefly summarize (in the next subsection) the main result on which we base our steady state analysis (a complete account can be found in [19]). In subsection 4.2 we apply this result to the present aquifer problem.

4.1 Steady state properties in a class of distributed control problems

We present the results in terms of the notations used in [19]. Consider a distributed control system where the state is represented by a function

$z(x,t)$ of the spatial variable x and the time $t \geq 0$, where $0 \leq x \leq L$ for some $L > 0$. The infinite-dimension state vector $\mathbf{z}(t)$ at time t, defined in terms of the function $x \mapsto z(x,t)$, belongs to a separable Hilbert space $\mathbf{H}$, and the infinite-dimension control vector $\mathbf{u}(t)$, defined in terms of $x \mapsto u(x,t)$, belongs to a separable Hilbert space $\mathbf{E}$.[4]

The time evolution equation describing the dynamics of the system is

$$\left(\sqrt{|z(x,t)|}\right)_t = (\alpha(x) z_x(x,t))_x + \gamma(x) z(x,t) + u(x,t), \tag{4.1}$$

for a continuously differentiable, positive function $\alpha(\cdot)$, and $\gamma(\cdot)$ is a continuous function which is either positive or negative on $[0,L]$.[5] We consider equation (4.1) with an initial condition

$$z(x,0) = \zeta(x),\ 0 \leq x \leq L, \tag{4.2}$$

$\zeta(\cdot)$ is differentiable, with boundary conditions of the form

$$a_1 z(0,t) + b_1 z_x(0,t) = \gamma_1(t),\ a_2 z(L,t) + b_2 z_x(L,t) = \gamma_2(t) \tag{4.3}$$

[4]It is further assumed that

$$\mathbf{H} = \mathcal{H}^1(0,L) = \{\xi(\cdot) \in \mathcal{L}^2(0,L) : \text{the derivative } \xi'(\cdot) \text{ belongs to } \mathcal{L}^2(0,L)\},$$

that the space $\mathbf{E}$ is $\mathcal{L}^2(0,L)$, and $\mathbf{u}(\cdot)$ is an $\mathcal{L}^2$ function from $[0,T]$ into $\mathbf{E}$, namely $u(\cdot,\cdot) \in \mathcal{L}^2((0,T),(0,L))$ (see details in [19]).

[5]This equation is related to the *porous medium equation*

$$z_t(x,t) = (z^m)_{xx},\ m > 1$$

and its generalization, the *filtration equation*,

$$z_t(x,t) = (\Phi(z))_{xx} + f(x,t).$$

For a discussion of these equations see [30].

for some constants a_1, a_2, b_1 and b_2, and for some continuous functions $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$ defined for $t \geq 0$.

Associated with the control system (4.1)-(4.3) is a payoff

$$J_T(\mathbf{z}(\mathbf{0}), \mathbf{u}(\cdot)) = \int_0^T F(\mathbf{z}(t), \mathbf{u}(t))dt, \tag{4.4}$$

defined for every $T > 0$, with

$$F(\mathbf{z}(t), \mathbf{u}(t)) = \int_0^L F_0(z(x,t), u(x,t))dx,$$

$F_0(z(x,t), u(x,t))$ being the instantaneous reward at location x and time t. The function $F(\cdot, \cdot)$ in (4.4) is concave and upper semi-continuous on $\mathbf{H} \times \mathbf{E}$, and satisfies the coercivity condition

$$F(\mathbf{z}, \mathbf{u}) \leq a - c||\mathbf{z}||^2_{\mathbf{H}} \tag{4.5}$$

for given constants $a > 0$ and $c > 0$.

The problem we deal with is infinite horizon so that the reward expressions J_T are to be maximized as $T \to \infty$. The optimality criterion that we adopt is the maximization of the long-run average reward, namely the maximization of

$$R(\mathbf{z}(\mathbf{0}), \mathbf{u}(\cdot)) = \limsup_{T \to \infty} \frac{1}{T} J_T(\mathbf{z}(\mathbf{0}), \mathbf{u}(\cdot)).$$

Definition 4.1 *A pair of functions* $(\mathbf{z}(\cdot), \mathbf{u}(\cdot)) : [0, \infty) \mapsto \mathbf{H} \times \mathbf{E}$ *is* admissible pair *if the following conditions hold:*

(i) $u(\cdot, \cdot) \in \mathcal{L}^\infty([0,T] \times [0,L])$ *for every finite* $T > 0$*, namely* u *is measurable*

and bounded on every finite rectangle $[0,T]\times[0,L]$.

(ii) $\mathbf{z}(\cdot):[0,\infty)\mapsto \mathbf{H}$ *is differentiable, and it satisfies* (4.1), (4.2), (4.3). *When considering the problem on the infinite time interval* $[0,\infty)$, *we assume that* $u\in\mathcal{L}^\infty([0,\infty)\times[0,L])$.

(iii) The function $t\mapsto F(\mathbf{z}(t),\mathbf{u}(t))$ *is locally Lebesgue integrable on* $[0,\infty)$.

Using the above terminology we say that $\mathbf{u}^\star$ is a *a good control* if

$$J_T(\mathbf{z}(0),\mathbf{u}^\star(\cdot)) > J_T(\mathbf{z}(\mathbf{0}),\mathbf{u}(\cdot)) - M$$

for some constant M, for every admissible control $\mathbf{u}(\cdot)$ and every $T>0$. The result below guarantees the existence of steady state for admissible good controls. Another notion which is needed to phrase the steady state result is the strong continuity with respect to the initial value and the non-homogeneous term (the control u) in (4.1).

Definition 4.2 *We say that the solutions of* (4.1) *depend strongly continuously on the initial value and the non-homogeneous term if the following holds. Suppose that* $\{\zeta_k\}_{k=1}^\infty$ *is a sequence of initial values that converge weakly in* $\mathcal{H}^1(0,L)$ *to a limit* ζ, *and for some fixed* $T>0$, *the sequence* $\{u_k\}_{k=1}^\infty$ *converges weakly in* $\mathcal{L}^2([0,L]\times[0,T])$ *to* u . *Let* $z_k(x,t)$ *be the solution of (4.1) corresponding to* ζ_k *and* u_k, *and let* $z(x,t)$ *be the solution corresponding to* ζ *and* u. *Then*

$$z_k(x,t)\to z(x,t) \text{ as } k\to\infty \tag{4.6}$$

point-wise for almost every $(x,t)\in[0,L]\times[0,T]$.

The following steady state result appears in [19, Theorem 3.4]:

Theorem 4.1 *Assume that F in (4.4) is strictly concave. Moreover, suppose that the solutions of (4.1) depend strongly continuously on the initial value and the non-homogeneous term. Let $(\mathbf{z}^\star(\cdot), \mathbf{u}^\star(\cdot))$ be an admissible and good pair, namely*

$$\int_0^T F(\mathbf{z}^\star(t), \mathbf{u}^\star(t))dt > \int_0^T F(\mathbf{z}(t), \mathbf{u}(t))dt - M \tag{4.7}$$

for some $M > 0$ and every admissible pair $(\mathbf{z}(t), \mathbf{u}(t))$. Then, there exists a steady state $\bar{\mathbf{z}}$ such that

$$\mathbf{z}^\star(t) \rightharpoonup \bar{\mathbf{z}} \text{ weakly as } t \to \infty. \tag{4.8}$$

4.2 Steady state properties of the aquifer problem

In applying the above result to the coastal aquifer problem we first need to show that (2.7) can be recast in the form of (4.1). Using (2.10) we express (2.7) as

$$\left(\sqrt{H(x,t)}\right)_t = \frac{(\kappa F(x) H_x(x,t))_x}{2F(x)\phi(x)} - \frac{b}{\phi(x)} H(x,t) + \frac{1}{\phi(x)}(a_0 - g(x,t)). \tag{4.9}$$

To show that (4.9) is equivalent to (4.1) use $z(x,t) = H(x,t)$ and $u(x,t) = (a_0 - g(x,t))/\phi(x)$ and note that, since $H(x,t)$ is nonnegative, we may consider $\sqrt{H}$ instead of $\sqrt{|H|}$ on the left-hand side. Defining

$$\xi(x) = 2\int_0^x F(s)\phi(s)ds$$

with the inverse $x(s) = \xi^{-1}(s)$, (4.9) is recast as

$$\left(\sqrt{|\tilde{z}|}\right)_t = (\tilde{\alpha}(\xi)\tilde{z}_\xi)_\xi + \tilde{\gamma}(\xi)z + \tilde{u}(\xi,t),$$

which has the same form as (4.1).

In fact, utilizing the present structure (particularly, Lemma 3.1), the following stronger property is verified in [19]:

Proposition 4.1 *Corresponding to the distributed control problem of maximizing* (3.17) *subject to* (2.7) *and the side conditions* (2.8)-(2.9)*, there exists a unique steady state to which any* good policy *converges in the long run.*

5 Characterization of the steady-state

In a steady state the various variables are time independent, so that if the reward expression is

$$R(T, g) = \int_0^T \int_0^L \mathcal{L}(x, H, g)\,dx\,dt \tag{5.1}$$

for an integrand $\mathcal{L}$ which is concave in (H, g) for every fixed x, then the steady state problem is to maximize

$$\hat{R}(\hat{g}) = \int_0^L \mathcal{L}(x, \hat{H}, \hat{g})\,dx \tag{5.2}$$

subject to (recalling (2.7) and (2.10))

$$\frac{\kappa}{2}\left[\hat{H}_{xx}(x) + \frac{F'(x)}{F(x)}\hat{H}_x(x)\right] + a_0 - b\hat{H}(x) - \hat{g}(x) = 0 \tag{5.3}$$

and side conditions, say the values of $\hat{H}$ and $\hat{H}_x$ both at $x = 0$ and $x = L$. For the special reward expression introduced above we have, recalling (3.18),

$$\mathcal{L}(x, \hat{H}, \hat{g}) = \hat{g}^{\gamma} - \frac{A(x)}{(\alpha\sqrt{\hat{H}} + \beta)^{\rho}}\hat{g}^{p}. \tag{5.4}$$

Using (5.3) we substitute

$$\hat{g}(x) = \frac{\kappa}{2}\left[\hat{H}_{xx}(x) + \frac{F'(x)}{F(x)}\hat{H}_x(x)\right] + a_0 - b\hat{H}(x) \tag{5.5}$$

in (5.2) and (5.4), obtaining a standard calculus of variations problem for the function $\hat{H}(x)$ on $0 \le x \le L$. The Euler-Lagrange equation for this problem is the following fourth order, boundary values ordinary differential equation (see, e.g., [7, Chapter 2])

$$\mathcal{L}_{\hat{H}} - b\mathcal{L}_{\hat{g}} - \frac{d}{dx}\left(\frac{\kappa F'(x)}{2F(x)}\mathcal{L}_{\hat{g}}\right) + \frac{\kappa}{2}\frac{d^2}{dx^2}\left(\mathcal{L}_{\hat{g}}\right) = 0 \tag{5.6}$$

where $\hat{H}(0) = 0$, $\hat{H}'(0)$, $\hat{H}(L)$ and $\hat{H}'(L)$ are prescribed boundary values. Thus, e.g., the term $\frac{d^2}{dx^2}\left(\mathcal{L}_{\hat{g}}\right)$ that appears in (5.6), involves the derivative of the function $\mathcal{L}(x, \hat{H}, \hat{g})$ with respect to its third variable $\hat{g}$, namely $\mathcal{L}_{\hat{g}}(x, \hat{H}, \hat{g})$, evaluated at the triplet $(x, \hat{H}(x), \hat{g}(x))$, where $\hat{g}$ is as in (5.5). This is a function of x alone, and its second derivative with respect to x is the quantity that appears in (5.6). This term yields expressions that include the fourth order derivative $\hat{H}''''(x)$, so that (5.5) is a non-linear, fourth order, boundary value equation.

6 Computation

It turns out that solving the Euler-Lagrange equation (5.6) is quite complicated, and the result thus obtained is not transparent. We will next take advantage of the structure of the problem, in particular the concavity of the integrand in (5.2), to offer a computation procedure of the steady state and the transition to it.

6.1 Steady state

The aquifer is divided into a small number of sub-regions N_1, N_2,...,N_m, where N_j is defined by

$$N_j = \{c_j \leq x \leq d_j\},\ 1 \leq j \leq m,$$

where $c_1 = 0$, $d_m = L$ and $d_j < c_{j+1}$, and where the following holds:

$$\frac{c_{j+1} - d_j}{L} << 1 \text{ for every } 1 \leq j \leq m - 1.$$

We suppose that within each region the characteristic feature of the aquifer (recharge $r(H(x))$, porosity $\phi(x)$, wells distribution $\lambda(x)$) are homogeneous and do not vary with x. Thus the recharge parameters $b(x)$ and $a_0(x)$ are labeled a_j and b_j according to the region they describe.

Our approach is to solve for the steady state within each region separately, and then combine all these solutions together to a steady state solution on the whole aquifer by considering the narrow strips

$$S_j = \{d_j \leq x \leq c_{j+1}\},\ 1 \leq j \leq m - 1$$

as boundary layers. However, since these strips are very narrow, and the system is actually discrete and not continuous, we don't have to be very precise as for the exact definition of the solution in these boundary layers small strips.

We next focus on a particular sub-region N_j. Let $\hat{g}(x)$ be a control with a corresponding state $\hat{H}(x)$, $c_j \leq x \leq d_j$, such that (5.3) holds in N_j. We denote the average values of these quantities

$$\bar{g} = \frac{1}{v_j}\int_{c_j}^{d_j} \hat{g}(x)F(x)dx, \; \bar{H} = \frac{1}{v_j}\int_{c_j}^{d_j} \hat{H}(x)F(x)dx, \tag{6.7}$$

where

$$v_j = \int_{c_j}^{d_j} F(x)dx$$

is the area of the region N_j. The reward expression associated with the pair $(\hat{g}, \hat{H})$ is

$$R_j(\hat{g}) = \int_{c_j}^{d_j} \left\{ \hat{g}^\gamma - \frac{A(x)}{(\alpha\sqrt{\hat{H}} + \beta)^\rho}\hat{g}^p \right\} F(x)dx, \tag{6.8}$$

where we suppress the dependence of R_j on $\hat{H}$ since $\hat{H}$ is determined by $\hat{g}$ via equation (5.3). We denote by $\mathcal{L}_j$ the restriction of $\mathcal{L}$ to N_j (recall (3.18)), and in view of the concavity of the integrand $\mathcal{L}_j(\hat{H}, \hat{g})$ in (6.8) it follows from Jensen inequality that

$$R_j(\hat{g}) \leq v_j \mathcal{L}_j(\bar{H}, \bar{g}). \tag{6.9}$$

This implies that the constant pair $(\bar{g}, \bar{H})$ yields a better reward than $(\hat{g}, \hat{H})$, provided that it is admissible, namely that it satisfies (5.3). But a constant pair (g_0, H_0) satisfies (5.3) if and only if

$$g_0 + bH_0 - a_0 = 0. \tag{6.10}$$

The non-constant pair $(\hat{g}, \hat{h})$, however, does satisfy (5.3), and integrating this equation on the interval $[c_j, d_j]$ and dividing the result by v_j yields

$$g_0 + bh_0 - a_0 = \frac{\kappa}{2v_j}[\hat{H}_x(d_j) - \hat{H}_x(c_j)]. \tag{6.11}$$

The right hand side of (6.11) is very small, since $\hat{H}_x = 2\hat{h}(x)\hat{h}'(x)$ and we have $\hat{h}(x) << F(x)$ and $\hat{h}'(x) \approx 0$ due to the assumption that N_j is homogeneous and depends weakly on x. Hence for any practical consideration the pair $(\bar{g}, \bar{h})$ may be considered as an admissible pair. Thus in view of (6.9), when addressing the maximization of (6.8) subject to (5.3) we may restrict attention only to constant pairs. Namely, the steady state maximizer of the problem on N_j is obtained as the solution to

$$\text{Maximize } \mathcal{L}_j(\hat{H}, \hat{g}) \text{ subject to } \hat{g} + b\hat{H} = a_0, \tag{6.12}$$

or equivalently, the solution of

$$\text{Maximize } \mathcal{L}_j(\hat{H}, a_0 - b\hat{H}) \text{ over all positive } \hat{H}. \tag{6.13}$$

The function $\mathcal{L}(\hat{H}, a_0 - b\hat{H})$ is a strictly concave function of $\hat{H}$, and it has a unique solution, which we denote $\hat{H}_j$ with a corresponding constant control

$$\hat{g}_j = a_0 - b\hat{H}_j.$$

We solve (6.12) (or (6.13)) for each region N_j, and define an admissible pair $(\hat{g}, \hat{H})$ on $0 \leq x \leq L$ as follows:

(i) On N_j we define

$$\hat{g} = \hat{g}_j \text{ and } \hat{H} = \hat{H}_j. \tag{6.14}$$

(ii) On S_j we choose any $\hat{H}_j(x)$ such that $\hat{H}_j$ is continuous at d_j and c_{j+1}, and then use (5.3) to define $\hat{g}_j(x)$. E.g., we may take $\hat{H}_j(x)$ as a third order polynomial such that in addition to the continuity at d_j and c_{j+1} it satisfies

$$\hat{H}_j'(c_j) = \hat{H}_j'(d_j) = 0.$$

Since the strips S_j are very narrow compared to the aquifer's length L, the exact definition of $(\hat{g}, \hat{H})$ on the strips S_j is not really significant, and we may consider the pair $(\hat{g}, \hat{H})$ defined in (6.14) and in (ii) above as the steady state optimal solution.

6.2 Transition to the steady state

We suppose that from a certain time T on the state approaches the steady state solution $(\hat{H}, \hat{g})$, and we have to solve for the transition period $0 \leq t \leq T$. We study this transition for each region N_j separately. Let $(g(x,t), H(x,t))$ be any pair satisfying (2.7) with recharge $r = a - bH$, and we consider this equation for $c_j \leq x \leq d_j$.

In this subsection we chose the state variable to be h rather than H. This simplifies the analysis, since when integrating (2.7), we obtain an equation for the average of h, with a small perturbation term, which may be ignored. This is equation (6.15) below. In this discussion of transition to steady state, it is more convenient to use the variable h, since then, under our assumptions, H practically disappears from the equation, and we are left with an equation that involves only the average of h, not averages of H or its derivatives.

Integrating (2.7) on this interval and dividing the result by v_j yields

$$\phi_j \bar{h}_j'(t) = \frac{\kappa}{2v_j}[F_j(H_x(d_j) - H_x(c_j))] + a_j - b_j \bar{h}_j(t) - \bar{g}_j(t) \tag{6.15}$$

where a_j and b_j are constants. As argued above, the term

$$\frac{\kappa}{2v_j}[F_j(H_x(d_j) - H_x(c_j))]$$

in (6.15) is very small, and we consider it as practically zero. We thus obtain the following dynamics on N_j:

$$\bar{h}'_j(t) = \alpha_j - \beta_j \bar{h}_j(t) - \delta_j \bar{g}_j(t) \tag{6.16}$$

where the constants α_j, β_j and δ_j are defined by

$$\alpha_j = \frac{a_j}{\phi_j},\ \beta_j = \frac{b_j}{\phi_j},\ \delta_j = \frac{1}{\phi_j}.$$

In addition, the following end conditions should be satisfied:

$$\bar{h}_j(0) = h_{0,j},\ h_j(T) = h^\star_j. \tag{6.17}$$

We have to maximize the reward (3.17) over all the admissible pairs (g_j, h_j), namely all the pairs that satisfy (6.16) and (6.17). Using again the concavity of $\mathcal{L}(g, h)$ and Jensen inequality, integrating (3.17) on $c_j \leq x \leq d_j$ for each fixed t, we obtain

$$R(T) \leq v_j \int_0^T \mathcal{L}(\bar{g}_j(t), \bar{h}_j(t))dt. \tag{6.18}$$

We compare an arbitrary admissible pair (g_j, h_j) to its associated average pair $(\bar{g}_j, \bar{h}_j)$, and in view of (6.18) we may focus on maximizing $\bar{R}(T)$ over admissible pairs which do not depend on the x variable. We are thus led to consider the maximization of

$$\bar{R}(T) = \int_0^T \mathcal{L}(\bar{g}_j(t), \bar{h}_j(t))dt \tag{6.19}$$

subject to (6.16) and the edge conditions (6.17). This is a standard optimal control problem, and we compute a solution by employing the Pontryagin

Maximum principle. We thus define the Hamiltonian

$$\mathcal{H}_j(g,h,\eta) = \mathcal{L}_j(g,h) + \eta(\alpha_j - \beta_j \bar{h} - \delta_j \bar{g}), \tag{6.20}$$

(recall (3.18)), and by the maximum principle $\eta_j(t)$ should satisfy the equation

$$\eta_j'(t) = -\frac{\partial \mathcal{H}_j}{\partial h}(\bar{g}_j, \bar{h}_j, \eta_j),\ 0 \le t \le T.$$

Namely

$$\eta_j'(t) = \beta_j \eta_j - \frac{\partial \mathcal{L}_j}{\partial h}(\bar{g}_j, \bar{h}_j). \tag{6.21}$$

Moreover, the optimal control $\bar{g}_j$ maximizes the function

$$g \mapsto \mathcal{H}(g, \bar{h}, \eta_j),$$

and in view of (3.18) it is the unique solution of

$$\gamma g^{\gamma-1} - pDg^{p-1} - \delta_j \eta = 0,$$

where

$$D = \frac{A_j}{(\alpha_j \bar{h}_j + \beta_j)^\rho}.$$

Thus $\bar{g}_j$ is the solution of

$$pDg^{p-\gamma} + \delta_j \eta g^{1-\gamma} - \gamma = 0, \tag{6.22}$$

which we denote by $\psi(\bar{h}_j, \eta_j)$. We substitute the control

$$\bar{g}_j(t) = \psi(\bar{h}_j(t), \eta_j(t))$$

in (6.16) and (6.21) and solve this system of equations subject to the boundary conditions (6.17)

This way we obtain for each region an optimal transition solution $(\bar{g}_j(t), \bar{h}_j(t))$ on $[0, T]$, which has the edge values $h_{0,j}$ and $\hat{h}_j$ in N_j. We then use some interpolation to define the transition solution on the strips S_j. Since these are very narrow compared to the length L, the exact definition of the transition control is not significant, and practically the definition we use yields an optimal transition solution.

7 Cunclusion

We analyzed dynamic-spatial management of a coastal aquifer under seawater intrusion using distributed control methods. We showed that the optimal policy converges over time to a spatially-dependent steady state function. This convergence property holds for a class of distributed control models which contains our coastal aquifer problem. The steady state function is obtained as solution of an ordinary control problem in space. Since analytical characterization of the steady state function and the transition to it are in general not available, we offered an approximation algorithm to calculate the optimal policy.

Our analysis abstracts in two important ways. First, the spatial variable in our model is single dimension, measuring the west-east distance from the coastal edge of the aquifer towards inland. Second, we assume a zero rate discounting. It would be of interest to consider a 2-dimensional space, by adding a variable that measures the north-south location, and to allow for a positive discount rate. These extensions are left for future research.

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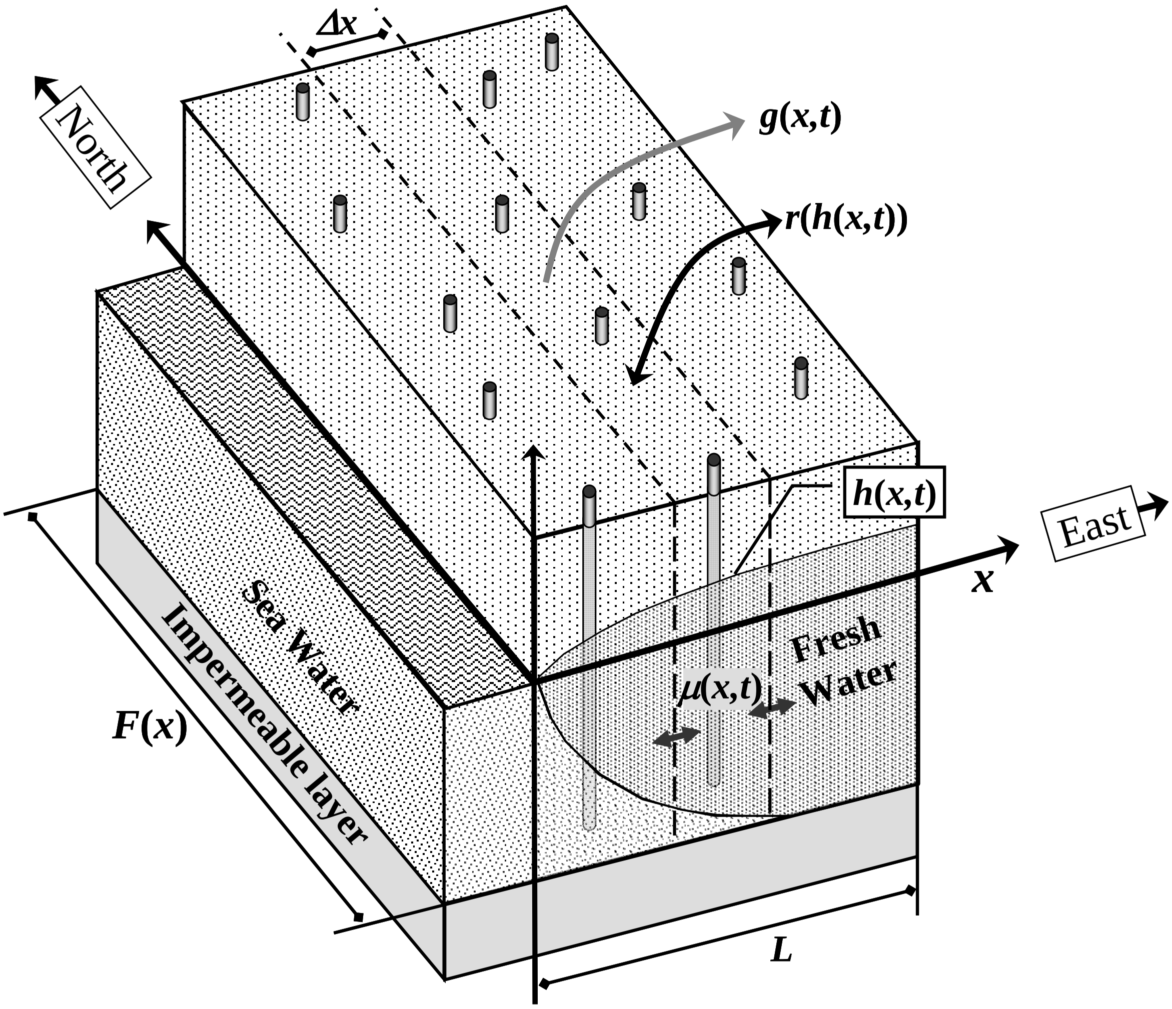

Figure 1. A 3-dimensional view of the coastal aquifer.

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