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MANAGEMENT, RISK AND COMPETITIVE EQUILIBRIUM

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1. Introduction

A farmer can vaccinate his livestock, install auxiliary irrigation systems, apply herbicides and pesticides, and in these ways significantly reduce the variability and uncertainty of the returns to his operation. More subtle, and of no lesser importance, is the ability to reduce risk by day-to-day management and observation. A good poultry grower will recognize a disease before it has spread in the flock and timely cultivation reduces the amount of weeds and their effect on crop yields.

In this paper I suggest a production model in which risk reduction is a function of managerial ability. This ability is not mean-preserving -- better management both reduces the variability of production and increases productivity (for an alternative specification see Pope and Just (1977)). The consequences of this ability to affect risk are analyzed in an industry characterized by a distribution of managerial abilities and perfect competition. Risk neutrality is assumed throughout. It will be shown that better managers will concentrate in the more risky activities -- realizing in this way their comparative advantage -- and that these activities will, as a result, project a relatively low risk image. The analysis is comparative static in nature, but I have in mind an economic selection process as the dynamic mover of the system. Accordingly, the risk considered is the risk associated with economic selection: of failing to cover costs and having to change lines of production. Competition and market forces, by reducing profit margins, increase this risk and tighten the selection stress.

2. Production and Skill Distribution

2.1 The Production Activity

Consider an agricultural industry producing a single product. All farms are of identical size and assume, for simplicity, that the level of input is the same on all farms. Let z be the dollar value of the constant, identical input vector. Since the analysis is long run in nature, z includes cost of capital services.

Assume that potential, maximal output in physical terms on each farm is θ units. Production is a random process and actual level of output is $q \leq \theta$. Assume that the probability distribution of the q values is the exponential density function (Figure 1) :

(1) $$f(q) = \eta e^{-\eta(\theta-q)} \qquad q \leq \theta$$

The cumulative distribution is

(2) $$F(q) = \int_{-\infty}^{q} \eta e^{-\eta(\theta-x)}dx = e^{-\eta(\theta-q)}$$

the expected value

(3) $$E(q) = \theta - 1/\eta$$

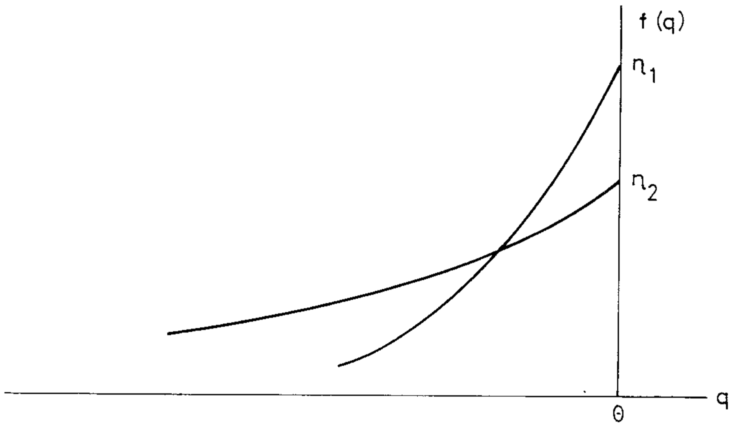

Figure 1: The exponential function $f(q) = \eta e^{-\eta(\theta-q)}$ for two values of η.

and the variance

(4) $\quad Var(q) = 1/\eta^2$

The parameter η is both the mean and the variance parameter.

Farmers differ in management ability. The symbol m stands for the management level and let $0 \leq m \leq 1$. To incorporate management into production, substitute in the distribution of outputs

$$\eta = \lambda m^\alpha \qquad 0 \leq m \leq 1,\ 0 < \alpha$$

Equation (1), for example, will be written as

(1') $\quad f(y) = \lambda m^\alpha e^{-\lambda m^\alpha(\theta-q)}$

At higher levels of m, the mean output, $E(q)$, will be higher and the variance will be lower. The parameter α was introduced to measure the intensity by which management can affect risk and productivity and will assume significance below in comparing lines of production.

With market price p the distribution of the dollar value of output, $y = pq$, is

(5) $\quad g(y) = \frac{\lambda m^\alpha}{p} e^{-\lambda m^\alpha(\theta-y/p)} \qquad y \leq p\theta$

$$G(y) = e^{-\lambda m^\alpha(\theta-y/p)}$$

with mean $p(\theta-1/\lambda m^\alpha)$ and variance $p^2/\lambda^2 m^{2\alpha}$.

A major measure of risk is the probability of negative profits. Operators for whom this probability is high, may lose often and will be forced to leave the industry. The probability of negative profits is, therefore, termed the selection stress.

Profits are

$$\pi = y - z$$

and the selection stress is

$$(6) \qquad Pr(\pi < 0) = \int_{-\infty}^{z} g(y)dy = G(z) = e^{-\lambda m^{\alpha}(\theta - z/p)}.$$

See Figure 2.

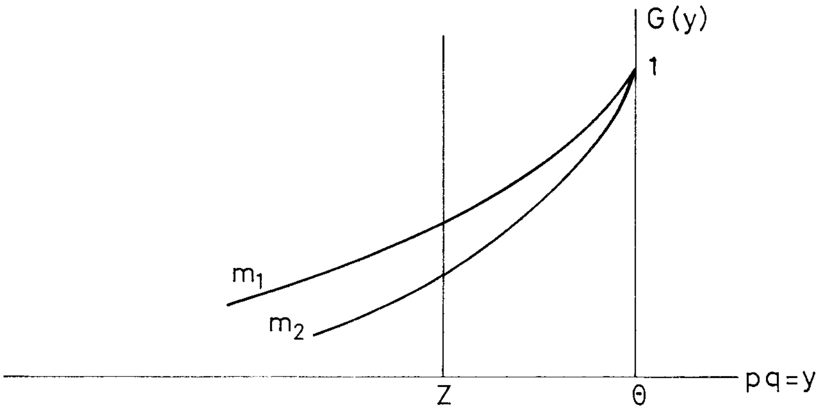

Figure 2: Risk, Pr(y<z), for two levels of management, $m_1 < m_2$.

Since both $\partial var(q)/\partial m$ and $\partial G(z)/\partial m$ are negative, management reduces the variability of outcomes of the production process and, thereby, reduces risk and the selection stress. A better manager faces, therefore, a smaller probability of failure.

2.2 The Industry

The industry is composed of operators of different managerial skills. Let N(m) be the number of operators with management level m, and assume the distribution of management abilities to be given by $N(m) = Am^{-\beta}$, $0<\beta<1$. To economize on symbols normalize by setting $A \equiv 1$ and write the distribution as

$$(7) \qquad N(m) = m^{-\beta}, \qquad 0<\beta<1, \; 0\leq m \leq 1.$$

The constraint on β reflects the assumption that the proportions of the management groups decrease with management level; see Figure 3.

Assume that the number of operators in the industry is large, so that N(m) can be taken as continuous in m. Let T_{ab} stand for the size of the group of operators with management abilities between m=a and m=b

$$(8) \qquad T_{ab} = \int_{a}^{b} N(m)\, dm = \frac{1}{1-\beta}(b^{1-\beta} - a^{1-\beta})$$

The total number of operators in the industry and outside is

$$(9) \qquad \int_{0}^{1} N(m)\, dm = \frac{1}{1-\beta}$$

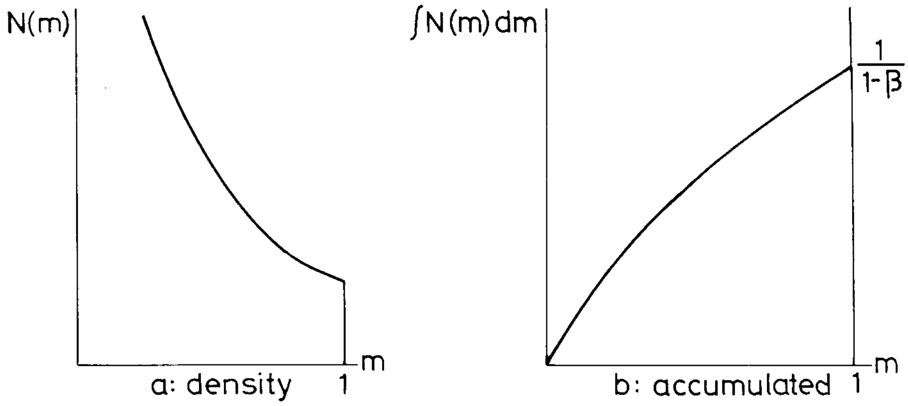

Figure 3: Distribution of Management

However, operators with relatively low levels of abilities cannot cover the cost of their production operations. Low level managers will, therefore, not be found operating in the industry.

Let $q_{.m}$ stand for the expected output of an operator with management level m. Total expected output for a group of firms, between the management levels a and b, is

(10) $$Q_{ab} = \int_a^b N(m) q_{.m} dm = \int_a^b m^{-\beta}(\theta - \frac{1}{\lambda m^{\alpha}})\, dm$$

Let $\delta \equiv 1-\beta-\alpha$, then for $\beta+\alpha \neq 1$,

(10a) $$Q_{ab} = T_{ab}\theta - \frac{1}{\lambda}\frac{b^{\delta} - a^{\delta}}{\delta}$$

and for $\beta + \alpha = 1$

(10b) $$Q_{ab} = T_{ab}\theta + \frac{1}{\lambda}(\log b - \log a)$$

Average, per firm, product in the group is

(11) $$Q_{ab}/T_{ab} \equiv Q..$$

If all operators with management abilities above the level m = a operate in the industry, b in equations (10) and (11) is replaced by 1.

Equation (12) specifies the variance of production in the industry as the sum of within firm and between firm variation.

(12) $$\sigma^2_{ab} = \int_a^b \frac{N(m)}{T_{ab}} \int_{-\infty}^{\theta} f(q)\,(q-q..)^2\, dq\, dm$$

$$= \frac{1}{T_{ab}} \int_a^b N(m) \left[\frac{1}{\lambda^2 m^{2\alpha}} + (q_{.m} - q_{..})^2 \right] dm$$

$$= \frac{1}{T_{ab}} \left[SW_{ab} + SB_{ab} \right]$$

The symbols SW_{ab} and SB_{ab} stand, respectively, for the within firms and between firms variability components. See Appendix for details.

The interpretation of this variance is the following: if repeated censuses (say, every year) of the group output were taken, and the variance of all the firm level observations around the long-run group average was calculated, its expected value would have been σ^2_{ab} as defined in (12). The specification in (12) does not assume independence of output in firms.

If operators in the industry are identical, output in each period can be regarded as a sample from the population of random outcomes whose variance is given by (12). This leads "naturally" to regarding the observed variability of output as a measure of the variance of the probability distribution facing each operator. Such a procedure, may be followed by a new operator contemplating entry or by an outside observer trying to assess uncertainty and risk associated with the industry (Rao, 1971). The same applies to weather related variability, if observations are taken over a period of years. However, even in agriculture, much of livestock, fruits, and vegetable production is quite independent of climatic changes, and still, as every producer is well aware, output variability, risk, and uncertainty are significant in these lines also.

In equation (12) the within firm variance, SW, depends on the management level, the between firm component -- on the degree of concentration of production along the skill axis. Thus, the higher the skill in an industry and the more concentrated its production, the lower the variance of output. A low variance industry may project the impression of a low-risk activity. This is the motivation for the analysis of the next section.

3. Comparative Advantage

3.1 Two Industries

Assume a production sector, say agriculture, composed of two industries: One producing product 1, and the other producing product 2. Let the demand functions be

(13) $p_i = c_i Q^{-\gamma}$ $\quad c_i, \gamma > 0, \quad i=1,2$

The management ability to affect the distribution of outcomes of production will now differ from industry to industry; index the parameter α, α_i $(i=1,2)$, in (1') and the equations that follow it.

We continue to assume an identical input vector of dollar value z in both industry 1 and 2. The output distributional parameters θ and λ are also identical, $\alpha_1 < \alpha_2$ (Figure 4). Demand may differ according to (13).

Recall the major assertion of the study; namely, that certain characteristics of the industrial organization -- particularly the variability of output and the terms of trade -- will differ in equilibrium configuration from what they otherwise may be. To demonstrate the effect of the market forces, conduct a "thought experiment:" in it, an imaginary configuration, state zero, which will be equilibrium state in all respects but one, will be compared to a final market equilibrium.

To define state zero assume, for simplicity, that market equilibrium can be maintained in each industry separately with identical number and skill distribution of producers. Thus, let the total number of operators with skill level m in the

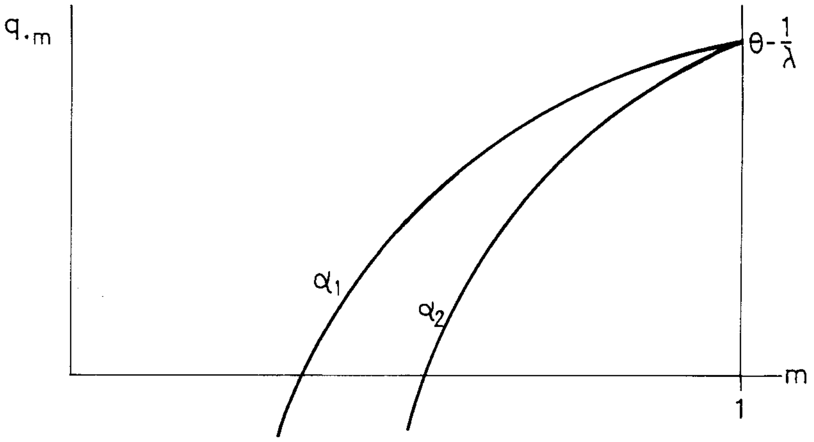

Figure 4: The functions $q_{.m} = \theta - \frac{1}{\lambda m^{\alpha_i}}$, $\alpha_1 < \alpha_2$

agricultural sector be $2m^{-\beta}$, of which $m^{-\beta}$ operate in each industry. Further, the skill of the marginal manager, the manager for whose firm revenue exactly equals cost, is the same in both industries. Mark this marginal skill level m_+, then

(14) $p_1 E_1(y) = p_2 E_2(y) = z, \qquad m = m_+$

See Figure 5. Operators with $m < m_+$ will, on the average, lose and will not produce. Total expected output of each product is

(15) $$Q_i = \int_{m_+}^{1} m^{-\beta} \left(\theta - \frac{1}{\lambda m^{\alpha_i}}\right) dm$$

$$= T_{+1}\theta + \frac{1}{\lambda} \frac{1-m_+^{\delta}}{\delta}, \quad \delta = 1-\beta-\alpha_i$$

according to (10a), assuming $\alpha+\beta\neq 1$. With these quantities, prices, p_1 and p_2, are determined in the markets according to (13).

Thus state zero is an equilibrium situation in most senses: product markets are in equilibrium, operating producers make profits, the marginal producers (of m_+) make zero profit, there are no losers in the industries considered. As will be seen momentarily, the only aspect in which the sector is not in equilibrium is the ordering of producers according to comparative advantage positions. But right now, at state zero, producers are distributed at random (i.e.uniformly) between the two industries.

3.2 Equilibrium and Comparative Statics

In state zero product 2 is more profitable than 1 (Figure 5). This is not an equilibrium situation; producers can improve their position by moving from product 1 to 2. Such a movement will reduce p_2 and increase p_1. In equilibrium it will not pay operators to shift production. Define π_{ki} as the profit of operator k in industry i. A producer in i cannot improve his position if for him

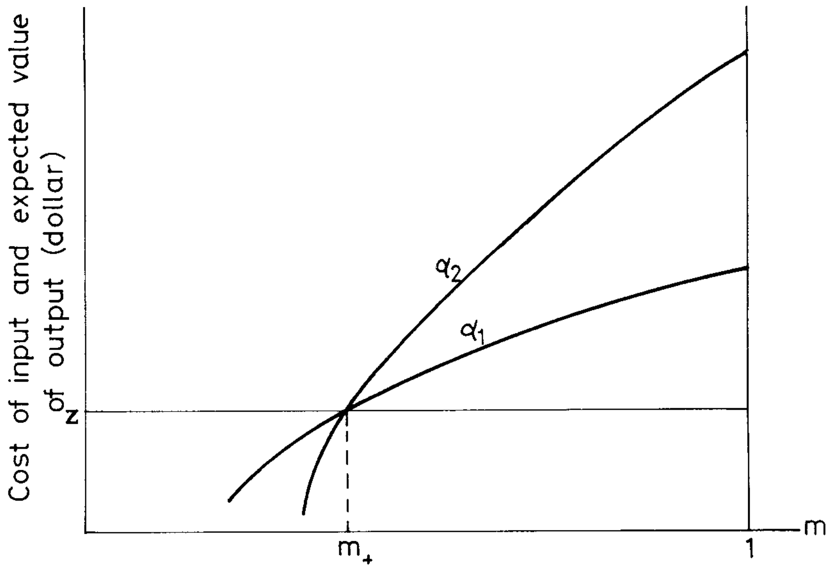

Figure 5: The functions $pq_{\cdot m}$ at state zero

$$E(\pi_{ki}) \geq E(\pi_{kj}) \qquad i, j=1,2$$

The sector is in equilibrium if the inequality holds for all k. An equilibrium is depicted in Figure 6.

In equilibrium, there are two break-even levels of management: at m_*

(16) $p_1E_1(q) = p_2E_2(q) \qquad m = m_*$

and m_* thus defines the boundary m -- farmers with $m < m_*$ produce product 1; those with $m_* < m$ produce 2. The second break-even point m_o is defined by zero profits

(17) $p_1E_1(q) = z \qquad m = m_o$

Producers with $m < m_o$ will not produce product 1; those with management ability on the range (m_o, m_*^o) will operate in industry 1. In Figure 6, $m_o < m_+$ -- the shift to ^{o} equilibrium called into production low management operators from other industries who could not have survived economically in state zero. The level m_o is defined by

(18) $z = p_1(\theta - \frac{1}{\lambda m_o^{\alpha_1}}) \qquad m_o = (\lambda(\theta - z/p_1))^{\frac{-1}{\alpha_1}}$

where p_1 is

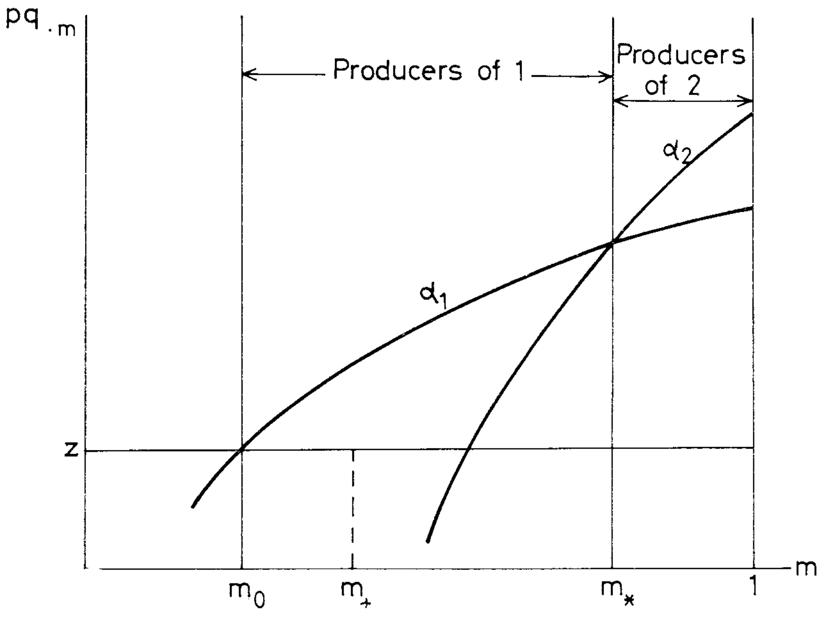

Figure 6: Equilibrium configuration of the agricultural sector

$$(19) \qquad p_1 = c_1 \left(\int_{m_o}^{m_*} 2m^{-\beta} \left(\theta - \frac{1}{\lambda m^{\alpha_1}}\right) dm \right)^{-\gamma} = c_1 Q_1^{-\gamma}$$

Note the factor 2 in the integrand in (19); it reflects the accumulation of producers from both industries. The same factor will apply similarly in the calculation of Q_2. (The integral in (19) assumes that potential producers of $m_o < m < m_+$ are also distributed according to $N(m) = 2m^{-\beta}$.)

By substituting m_o from equation (18) into G(y) in (5) one finds that for m_o the selection stress is $e^{-1} = .37$. The marginal producer will break even on the average, he will lose a third of the time and make profits 2/3 of the time.

In a dynamic environment, with farmers entering into and exiting from lines of activity, the selection stress is interpreted as the probability that a producer, chosen at random, will attempt to enter an industry, lose and fail. Comparing the equilibrium to state zero, we note, that since p_2 is lower and p_1 is higher the selection stress is, in equilibrium, tighter in industry 2 and looser in industry 1 than in state zero.

The shift from state zero to equilibrium also changed riskiness, as defined in equation (6). It is now more risky for a relatively low m farmer to move from product 1 to 2. In equation (6) for given m and α

$$\frac{\partial G(z)}{\partial p} < 0$$

The changes in the terms of trade made industry 1 less risky and industry 2 more risky than in state zero.

The observed variance, as defined in equation (12), also differs in equilibrium from the state zero variance. In industry 2, equilibrium variance is clearly lower than state zero variance -- both within firms and between firms variances are smaller. The reduction in the variance in the value of the output is even larger since p_2 is small at equilibrium than in state zero.

It is probable that the observed variance in industry 1 will grow with the shift from state zero to equilibrium -- within firms variance grows and p_1 rises-- but since the variance between firms may be smaller, this conclusion cannot be general.

4. A Numerical Example

Consider 2 industries with the following common parameters:

$$\theta = 8$$
$$\lambda = 1$$
$$\gamma = 1$$
$$\beta = 0.5$$
$$z = 4.38$$

The industry-specific parameters are:

	α_i	α_i
Industry 1	0.8	6.78
Industry 2	1.2	23.31

With these specifications m_+, the break-even point for both industries at state zero, is 0.2 with prices:

$$p_1 = 1.00$$
$$p_2 = 3.97$$

and $p_1E_1(q) = p_2E_2(q) = z = 4.38$. See the solid lines in Figure 7.

To simplify the calculations, I assumed in this numerical example that m_+ will also in equilibrium be the lower bound management level. That is, new operators will not enter the industry even if profits are positive for a range of management level lower than m.

The second equilibrium break-even point is $m_* = 0.3742$. This is the dividing management level between the equilibrium allocation of producers to industries 1 and 2. See the broken lines in Figure 7.

Figure 8 depicts standard deviation of dollar value of output for both industries, $p_i m^{-\alpha_i}$, for state zero (solid lines) and for equilibrium (broken lines).

Table 1 presents a set of selected results of the numerical example. The reading of the table can be exemplified with the average product variable (q..). At state zero the average product per operating farm in industry 1 is 6.13; the same variable assumes the value of 5.19 in equilibrium. Per-farm product is lower in equilibrium; it is only 84 percent of the state zero level. On the other hand, the equilibrium level of industry 2 is 116 percent of the state zero average product of that industry.

The magnitudes reported in Table 1 illustrate well, I trust, the theoretical analysis of the earlier sections of the paper. Since their meaning has mostly been discussed at length, I am leaving the detailed examination and interpretation of the table to the interested reader.

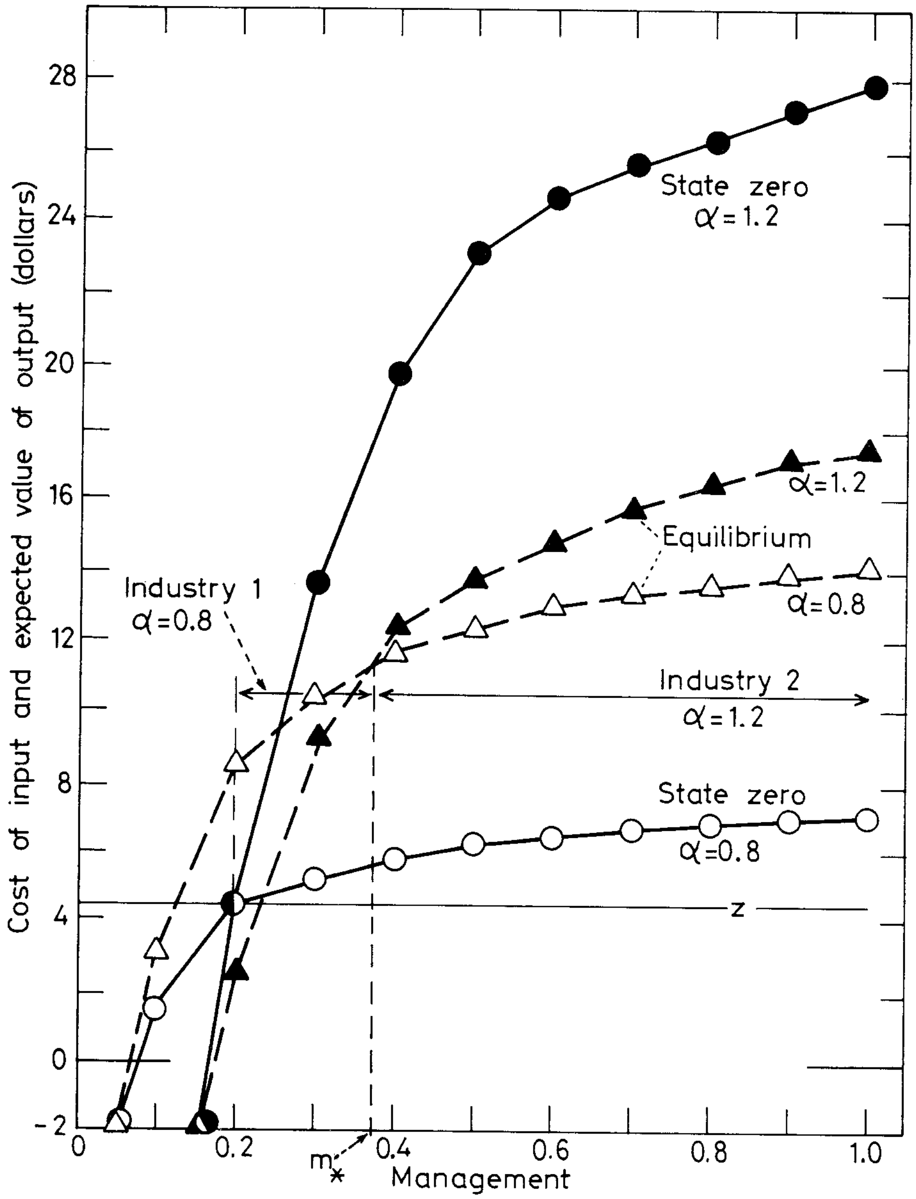

Figure 7: Value of output as a function of management

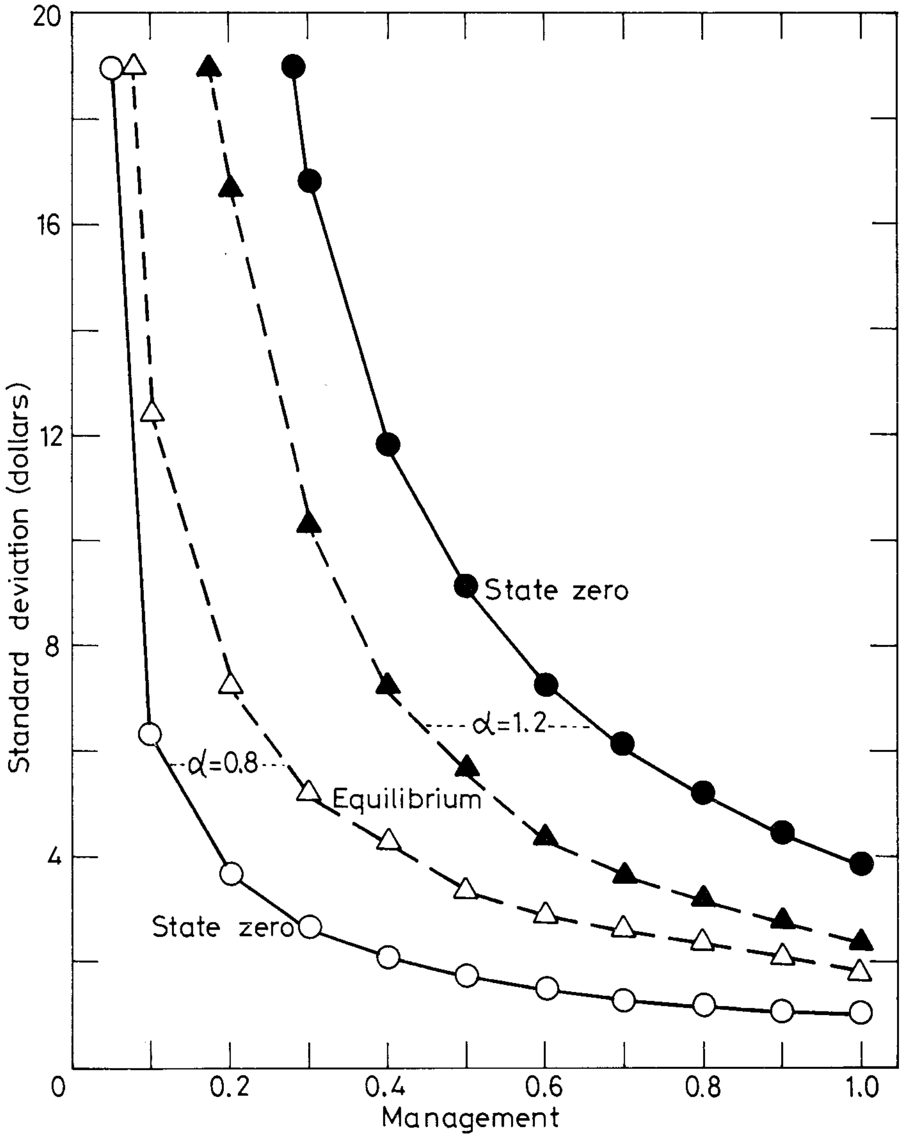

Figure 8: Standard deviation of value of output

Table 1: The numerical example - comparative statics

	State zero		Equilibrium		Equilibrium state zero	
	1	2	1	2	1	2
Market price (p)	1.00	3.97	1.98	2.43	1.98	0.61
Industry's output (Q)	6.78	5.87	3.41	9.60	0.50	1.64
Average product ($q_{..}$)	6.13	5.31	5.19	6.18	0.84	1.16
Number of producers (T)	1.11	1.11	0.66	1.55	0.60	1.40
Profits at average output ($\pi = pq_{..} - z$)	1.75	16.71	5.87	10.63	3.63	0.64
Variance within firms (SW/T)	4.01	9.66	8.08	3.71	2.02	0.38
Variance between firms (SW/T)	0.51	2.40	0.17	0.39	0.33	0.16
Industry variance (σ^2)	4.52	12.06	8.25	4.10	1.83	0.34
Industry-wide standard deviation of output ($p\sigma$)	2.12	13.79	5.67	4.92	1.38	0.36
Average management[a] ($m_{..}$)	0.46	0.44	0.27	0.61	0.60	1.39
Selection Stress						
Probability of loss at $m_{..}$	0.14	3.8×10^{-5}	0.02	2.5×10^{-4}	0.14	6.58
Probability of loss at[b] E(m)	0.22	0.16	0.09	0.19	0.41	1.19

[a] $m_{..}$ defined by $q_{..} = \theta - 1/\lambda m_{..}^{\alpha}$.

[b] $E(m) = \int_0^1 N(m) m\,dm = \frac{1-\beta}{2-\beta}$

5. Concluding Remarks

Most economic discussions of risk assume a given, subjective or objective, variability in production and returns and analyze the behavior of economic agents in terms of decision theory and readiness to accept risk. An economic unit is assumed to be able to affect its total risk position by selecting portfolios of venture but otherwise it accepts passively whatever riskiness nature offers. Perhaps typically, Arrow's (1971) book deals with risk bearing. Operations research applications have followed the same lines.

The first purpose of this paper was to draw attention to the managerial ability to affect risk and to its economic consequences. But the moral of that story is of wider implications: it means that subjective assessment of the world (subjective probabilities) and capricious preferences (utility) are, in a competitive environment, restricted by technology and market forces. This seems often to have been neglected (for example, by Anderson, Dillon and Hardacker (1977) and by Lin Deal and Moore (1974), but not by Roumasset (1974)). The analysis is also presented as a contribution toward the construction of a theory of economic evolution (Alchian (1950)), which will have though, by its very nature, to be a dynamic theory.

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APPENDIX: Industry wide variance of production.

(A.1) $$\sigma^2_{ab} = \int_a^b \frac{N(m)}{T_{ab}} \int_{-\infty}^{\theta} f(q)\ (q - q_{..})^2 dy\ dm$$

$$= \int \frac{N(m)}{T_{ab}} \int_{-\infty}^{\theta} f(q)[(q-q_{.m})^2 + (q_{.m}-q_{..})^2 + 2(q-q_{.m})(q_{.m}-q_{..})]\ dq\ dm$$

$$= \frac{1}{T_{ab}} [SW_{ab}+SB_{ab}] + 2\int_a^b \frac{N(m)}{T_{ab}} \int_{-\infty}^{\theta} f(q)[qq_{.m}-qq_{..} - q^2_{.m} + q_{.m}q_{..}]\ dq\ dm$$

$$= \frac{1}{T_{ab}} [SW_{ab} + SB_{ab}]$$

(A.2) $$SW_{ab} = \int_a^b N(m) \int_{-\infty}^{\theta} (q-q_{.m})^2\ dq\ dm$$

$$= \int_a^b m^{-\beta} (\frac{1}{\lambda^2 m^{2\alpha}})\ dm$$

$$= \frac{1}{\lambda^2} \frac{b^\xi - a^\xi}{\xi} \qquad \beta + 2\alpha \neq 1$$

$$= \frac{1}{\lambda^2} (\log b - \log a) \qquad \beta + 2\alpha = 1$$

$$\xi = 1 - \beta - 2\alpha$$

(A.3) $$SB_{ab} = \int_a^b m^{-\beta} (y_{.m} - y_{..})^2\ dm$$

Define

$$\delta = 1 - \beta - \alpha$$

$$A \equiv \frac{b^\delta - a^\delta}{\delta} \qquad \beta + \alpha \neq 1$$

$$\equiv \log b - \log a \qquad \beta + \alpha = 1$$

(A.4) $$SB_{ab} = \frac{1}{\lambda^2} (\frac{b^\xi - a^\xi}{\xi} - \frac{A^2}{T_{ab}}) \qquad \beta + 2\alpha \neq 1$$

$$= \frac{1}{\lambda^2} (\log b - \log a - \frac{A^2}{T_{ab}}) \qquad \beta + 2\alpha = 1$$

Two cases apply if β plus $2\alpha \neq 1$, either $\beta + \alpha \neq 1$ or $\beta + \alpha = 1$.