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A SIMPLE MODEL OF SENIORITY AND TURNOVER

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Abstract

A two period model in which workers are trained before commencing employment is considered. Wages are set to reduce turnover and cost of training. It is shown that in the absence of discounting, wages will rise with seniority, but this need not be so if rates of interest are positive. It is also shown that the wage paid in the second period may exceed marginal product. The main results hold in a three period model and extensions to n periods are considered. The discussion stresses exposition and intuitive explanation.

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A SIMPLE MODEL OF SENIORITY AND TURNOVER

Hiring workers and training them for their tasks is an investment; often in firm specific human capital. The investment creates an incentive for the employer to reduce turnover by increasing the salaries of the employees. Two patterns of investment were suggested: on the job training that reduces the contribution of the worker at the training period and increases it afterwards, and the simpler alternative of hiring cost and entry point training--before the commencement of employment. Similarly, premiums to workers with firm specific capital were suggested both as seniority scales and as higher flat rate salary. The economics of on the job training was elaborated on by Becker (1975); Stiglitz (1974) assumed entry point training and higher pay in his model of urban unemployment in developing countries. Recently Collier and Knight (1986) suggested a model of entry point training to explain seniority premium in the United Kingdom and Japan. A concise review of recent literature is included in Yellen (1984); Mazumdar (1983) presents a useful discussion of urban labor markets in LDCs.

Seniority premiums were explained as payments for accumulated experience, a means for selection of prospective loyal employees, an incentive for effort, a way to share investment in human capital, and a form of an optimal contract. The determination of a wage structure aimed at reducing quitting and saving on hiring cost is perhaps the simplest case to analyze,

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but the algebra of even a simple problem of turnover optimization is quite complicated. I wish to show that the analysis of such a problem is not only complicated but also illuminating. The discussion in the paper is restricted to a two period problem, but the main findings are extended to three periods. Hiring and training costs are incurred only at the entry point--on the day of recruitment (sometimes called period zero). It is shown that in this model, as long as the discount factor is unity (zero interest)--the wage scale will be rising, with the wage paid in the second year higher than in the first. The wage scale does not necessarily rise, and it may even decline, when the interest rates are positive. It is also shown that the wage paid in the second period may be higher than the marginal product of labor since the benefits to the firm of the worker staying for another period come both in product and in reduced training. We conclude that the model developed may indeed explain seniority premium, but it is not likely that its underlying assumptions are the sole explanation. Given the richness and the complexity of labor relations, this should not come as a surprising conclusion.

The discussion follows Collier and Knight's. The definition of the problem and some of the analytical tools are borrowed from their presentation. The findings, however, are not all the same; but the paper is not written as a systematic criticism of the Collier and Knight analysis.

Preliminaries

To make the notion of a 2-period analysis clear and precise, we make the following assumptions. A firm hires workers on December 31 each year. The workers are identical. Once hired, they are trained at a cost T to the firm and thus acquire firm specific capital. The workers do not invest time or money in training. Hiring is for two years, but some workers quit before the mandatory retirement date: either on January 1 of the first year, with probability q_1, or on January 1 of the second year, with probability q_2. Those who do not quit on the first of the year, stay to its end.

The quit rates, q_i, are functions of the wages[1/]

$$(1) \quad q_1 = q_1(U(w_1) + U(w_2))$$

$$q_2 = q_2U(w_2)$$

$$0<q_i<1, \ q'_i()<0, \ q''_i()>0 \quad i = 1,2$$

where w_1, w_2 are, respectively, wage in year 1 and in year 2 in the firm; $U(w_i)$ is the utility function, with the usual concavity properties.

The assumption in equation (1) is that the workers do not have access to the capital market, their rate of discount is zero and they consume all their earnings, wages, at the period at which they receive them. With these assumptions, the utility of future incomes is simply the sum of future utilities. The quitting rate is a decreasing function of the utility of staying with the firm. Implicitly and in the background, the worker is also affected by the utility of alternative employment. We are not analyzing the details of the quitting decision and take a somewhat mechanical approach--workers act as if their behavior reflects a random process with probability of quitting affected by the wage schedule.

The function q_1 is simplified in yet another way: in considering whether to stay with the firm, a rational individual will generally also take

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into account that he may quit at the beginning of the second year. Therefore, q_2 should appear as an argument in $q_1()$. This possibility is disregarded in equation (1). Non-zero interest rates and expected future quits are introduced below.

We consider the firm with a given number of workers, E, and at the steady state. The firm recruits each year RE workers (R is the percentage share of new recruits). The workforce cohorts are

(2) $E_1 = ER\,(1-q_1)$

$E_2 = ER\,(1-q_1)\,(1-q_2)$

The total number of workers

(3) $E = E_1 + E_2$

from which the steady state recruitment rate is

(4) $R = [(1-q_1) + (1-q_1)(1-q_2)]^{-1}$

Given the size of the labor force, the firm's objective is to minimize cost per worker, including cost of training T,

(5) $z = R\,[T + w_1(1-q_1) + w_2(1-q_1)(1-q_2)]$

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The control variables are w_1 and w_2 and the first question is: is seniority premium, $w_2 > w_1$, optimal?

Structure

It is shown in the Appendix that optimality of seniority premium can be proved, in this model, in a straightforward manner using Lagrange constrained minimization. Here we take an expository approach to the problem and examine its separate components. Start by minimizing z in equation (5)

$$(6) \quad \frac{dz}{dw_1} = \frac{\partial z}{\partial q_1} \frac{\partial q_1}{\partial w_1} + \frac{\partial z}{\partial w_1} = 0$$

$$\frac{dz}{dw_2} = \frac{\partial z}{\partial q_1} \frac{\partial q_1}{\partial w_2} + \frac{\partial z}{\partial q_2} \frac{\partial q_2}{\partial w_2} + \frac{\partial z}{\partial w_2} = 0$$

Second order conditions are assumed to be satisfied.

The first partials are the effects of changes in the quit rates on labor cost

$$(7) \quad \frac{\partial z}{\partial q_1} = \frac{\partial R}{\partial q_1} [\quad] - R\left[w_1 + w_2 (1-q_2)\right]$$

$$\frac{\partial z}{\partial q_2} = \frac{\partial R}{\partial q_2} [\quad] - R(1-q_1)w_2$$

The effects of changes in the quitting rates on the rate of recruitment--the derivatives in the right hand side of eqs. (7)--are expressed in the following:

$$(8) \quad \frac{\partial R}{\partial q_1} = R^2(2-q_2)$$

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$$\frac{\partial R}{\partial q_2} = R^2(1-q_1)$$

Rearranging (7), incorporating (8),

$$(9) \quad \frac{\partial z}{\partial q_1} = R^2(2-q_2)T$$

$$\frac{\partial z}{\partial q_2} = R^2(1-q_1)T + R^2(1-q_1)^2(w_1-w_2)$$

$$= \text{(training cost effect)} + \text{(wage rate effect)}$$

An increase in either q_1 or q_2 raises the recruitment rate and therefore the total training cost of the firm . The training cost effect of a change in q_1 is approximately double that of a change in $q_2$$\left[(2-q_2)\geq 2(1-q_1) \text{ if } q_2\leq 2q_1\right]$ because a laborer quitting at the beginning of the first year creates a two cohort vacancy to be filled by new trainees. At the same time, a change in q_1--the rate of quitting before ever starting work--does not affect the cohort-seniority distribution and does not affect, therefore, the wage bill of the firm. A change in q_2 changes the cohort distribution and, if $w_1 \neq w_2$, changes total wage payment.

The first order conditions derived from equations (6) can now be written as

$$(10) \quad - R^2T(2-q_2)\frac{\partial q_1}{\partial w_1} = R(1-q_1)$$

$$- R^2T\left[(2-q_2)\frac{\partial q_1}{\partial w_2} + (1-q_1)\frac{\partial q_2}{\partial w_2}\right] - R^2(1-q_1)^2(w_1-w_2)\frac{\partial q_2}{\partial w_2} = R(1-q_1)(1-q_2)$$

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In equations (10) the terms with R^2T are training cost effects; the others are wage bill effects of changes in w_1 and in w_2.

Interpretation

There are two aspects to the optimization of the wage rate: the wage level and the seniority scale. In this section we consider the wage level. To this end assume that the firm seeks to optimize wage payments while maintaining $w_1 = w_2$. In this case, the term with $w_1 - w_2$ vanishes--there is no cohort distribution effect--and the marginal benefit of a change in the wage level is the sum of the left-hand-sides in (10). The sum of the right-hand-sides is 1, this is the marginal cost of increasing the wage level by one unit (of adding 1 to the wage of the representative worker who is a weighted average of the two cohorts).

Seniority Premium

It will be useful to start with an intuitive argument--to be made rigorous below. We have seen that the training cost effect of a change in q_1 is approximately twice the size of a similar change in q_2. From this perspective, it is more important for the firm to reduce q_1 than to reduce q_2. The most effective way to reduce q_1 is to increase wage payment while maintaining $w_1 = w_2$, as considered in the earlier section. This is the most effective way because the argument in $q_1()$ is the sum $U(w_1) + U(w_2)$ and any departure from equal wages reduces utility due to the concavity of the utility function $U()$. Nevertheless, with the present formulation, a seniority premium will be preferred.

To see why wages will rise with seniority, consider a firm which

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minimized z in (5) subject to $w_1 = w_2 = \bar{w}$. Let the seniority pay be x (small) such that $w_2 = w_1 + x$. From the constraint equilibrium the firm moves to a seniority pay regime by setting $w_1 = \bar{w} - x/2$, $w_2 = \bar{w} + x/2$. Such a change will have two desirable effects on the firm: it will reduce total wage bill, because the first cohort is larger than the second, and it will reduce q_2. In general, a move to seniority pay will also have an undesired effect--the aforementioned reduction in first period utility and, consequently, increased quitting. However, at the constrained equilibrium point at which $w_1 = w_2$, this undesired effect is negligible for small values of x.

More rigorously, we wish to show that $\partial z/\partial x > 0$, which in turn implies that (11) holds.

$$(11) \quad \frac{dz}{dw_1} > \frac{dz}{dw_2} \quad \text{for minimum z, subject to } w_1 = w_2$$

It also means

$$(12) \quad \frac{dz}{dw_1} > 0 \qquad \frac{dz}{dw_2} < 0$$

The equality $w_1 = w_2$ implies by eq. (1), $\partial q_1/\partial w_1 = \partial q_1/\partial w_2$, and that (11) can be rewritten, using (10),

$$(11') \qquad R(1-q_1) > R^2T(1-q_1)\frac{\partial q_2}{\partial w_2} + R\ (1-q_1)(1-q_2)$$

$$q_2 > RT\frac{\partial q_2}{\partial w_2}$$

The last inequality always holds because $\partial q_2/\partial w_2 < 0$. This proves that with the current assumptions a seniority premium is preferred to equal pay.

Time Preference

The firm and the workers are affected differently by the introduction of discount factors, even if their subjective rates of interest are identical.

Focusing on seniority premium, we may disregard non-labor capital. For concreteness, assume that workers are paid each December 31 after a year of work. A worker who works to retirement receives two salary payments. The procedure of paying once annually at the end of the year is followed by all employers in the economy. We may, thus, disregard discounting in the first year and write the utility upon entering employment in the firm as

$$U_o = U(w_1) + \alpha\, U(w_2)$$

with α the discount factor $(0<\alpha<1)$. The quit function $q_1()$ is now

$$(13) \quad q_1 = q_1\left[U(w_1) + \alpha U(w_2)\right]$$

As for the firm, its monetary outlays are training costs and wages on December 31 of each year. To maintain the symmetry of the basic conditions of the workers and the firm, assume that the firm faces the same interest rate as the workers and that its time schedule is also similar to that of its laborers: let it sells its product on December 30 of each year. So the firm has to borrow the total labor cost each year for one year (or it uses equity capital, the alternative cost of which is the common rate of interest). The

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firm's objective is to minimize z/α,

$$(14) \quad z/\alpha = R\left[T + w_1(1-q_1) + w_2(1-q_1)(1-q_2)\right]/\alpha$$

where $q_1()$ is defined by (13) and $q_2()$ is still defined as in (1). The controls are again, w_1 and w_2.

Equations (13) and (14) demonstrate the difference in the effect of discounting on the firm and on the worker. For the worker, a seniority scale means deferred payments. The firm is only affected by the total cost of labor. Inter-cohort shifts of wages, so long as they do not affect total labor cost, do not affect directly the firm's cost (indirectly they do, by modifying quit rates).

Given α (a constant), minimizing z/α is minimizing z in (14). Start the analysis, again, from a minimization constrained to $w_1 = w_2$. Noting that

$$(15) \quad \frac{\partial q_1}{\partial w_2} = \frac{\partial q_1}{\partial w_1}\alpha \quad \text{for } w_1 = w_2$$

equation (11') can be rewritten as

$$(16) \qquad R^2T(2-q_2)\frac{\partial q_1}{\partial w_1}(1-\alpha) + R(1-q_1) > R^2T(1-q_1)\frac{\partial q_2}{\partial w_2} + R(1-q_1)(1-q_2)$$

$$RT\frac{\partial q_1}{\partial w_1}\left(\frac{2-q_2}{1-q_1}\right)(1-\alpha) + 1 > RT\frac{\partial q_2}{\partial w_2} + (1-q_2)$$

$$q_2 > RT\left[\frac{\partial q_2}{\partial w_2} - \frac{\partial q_1}{\partial w_1}\left(\frac{2-q_2}{1-q_1}\right)(1-\alpha)\right]$$

Since both $\partial q_2/\partial w_2$ and $\partial q_1/\partial w_1$ are negative, the right hand side in the last line of (16) may be positive and then the inequality will hold only for relatively large values of q_2. With positive interest rates, with $0 < \alpha < 1$, seniority premium is not always optimal (but note that for $\alpha = 1$, eq. (16) is reduced to (11')).

The machinery of the analysis can be used to rescue a quasi seniority premium. Let the firm minimize z subject to the constraint

$$(17)\quad \frac{\partial q_1}{\partial w_1} = \frac{\partial q_1}{\partial w_2}$$

Equation (17) implies $w_1 > w_2$, and given (17) the inequality in (11') holds. Hence, at the optimum, w_2 will always be higher (relative to w_1) than the level needed to maintain (17). This is the sense in which a quasi premium will prevail even with a positive discount rate. But only if the inequality in (16) holds, will an actual seniority premium, $w_2 > w_1$, be optimal.

Moreover, if the inequality in (16) is reversed, the optimal pay scale will be $w_1 > w_2$. With positive discount rates, negative seniority premiums are also likely.

Marginal Product

In the original Becker analysis, a worker receives more than the marginal product in the first period with the firm--the training period--and receives less than the marginal product thereafter.[2/] In the present model, there is no training period and the issue of the relation between the wages and the marginal product did not arise because we have hitherto assumed

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constant marginal product and constant workforce. Under these assumptions, the decision of the firm is divided into two stages: (a) minimize z in (5) or in (14); (b) if z<m (m being the value of the marginal product) do not operate the firm, if z>m--operate.

It is quite simple analytically to incorporate in the discussion a production function with decreasing marginal productivity of labor and to let the firm expand employment until the marginal cost of labor equals its marginal product. However, since the algebra is cumbersome, and we are only interested in finding whether $w_2 > m$ is possible, we adopt a simplified approach.

Assume that at the constrained minimum ($w_1 = w_2 = \bar{w}$) of equation (5) the solution is such that $z = m$: on its labor account the firm just breaks even. At this point, $z = RT + \bar{w}$. Disregarding for the moment changes in q_i and hence changes in R, if the firm sets the seniority premium $x \geq 2RT$, then $w_2 = \bar{w} + x/2 \geq m$.

We cannot repeat now the steps that led to the inequality (11') since that analysis was appropriate only for small values of x. For large values of x, the worker has to be compensated for departure from $w_1 = w_2$. The compensation, C, similar to risk premium, is approximated by the following expression[3/]

$$(18)\quad C = -\left(\frac{x}{2}\right)^2 \frac{U''(\bar{w})}{U'(\bar{w})}$$

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Let x = 2RT, labor cost per laborer is then

$$(19)\quad z = RT + R\left[(\bar{w}-RT)(1-q_1) + (\bar{w} + RT)(1-q_1)(1-q_2)\right] - (RT)^2 \frac{U''(\bar{w})}{U'(\bar{w})}$$

$$= RT + \bar{w} - R^2T(1-q_1)\, q_2 - (RT)^2 \frac{U''(\bar{w})}{U'(\bar{w})}$$

If $x \geq 2RT$, second period payment exceeds marginal product. We show now that such a value of x may be optimal. Recall that in the absence of a seniority premium $z = RT + \bar{w}$. Hence, the sufficient condition that a seniority premium will be larger than 2RT and $w_2>m$ is that z in (19) is smaller than $RT + \bar{w}$, which in turn implies

$$(20)\quad R^2T(1-q_1)q_2 > \left| (RT)^2 \frac{U''(\bar{w})}{U'(\bar{w})} \right|$$

$$\frac{(1-q_1)\, q_2}{T} > \left| \frac{U''(\bar{w})}{U'(\bar{w})} \right|$$

The inequality in (20) is only a sufficient and not a necessary condition because we have disregarded the beneficial effect of the seniority pay in reducing q_2. Therefore, $w_2>m$ is possible even if (20) does not hold; but if (20) holds--the wage rate in the second period will be higher than the value of the marginal product of labor.

The last conclusion was reached under the restrictive assumptions that E=constant and z=m. Instead, let now the size of the workforce vary, and

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assume that marginal productivity of labor is decreasing, and that marginal cost of each laborer is z (changes in the length of the work day or the working week are not possible). Then, in equilibrium z=m, and $w_2>m$ if (20) holds.

There is no apparent reason why the inequality in (20) should never hold. We conclude therefore that it is in principle possible that the second period wage will be higher than the marginal product.

Extensions

We have seen that a rising wage scale in a firm with two period employment reduces turnover and shifts wages from the larger to the smaller cohort. The same is true for a three cohort firm. Proof of the optimality of seniority payments in a three period model is outlined in Part B of the Appendix. In principle, there is no reason why the same considerations will not apply in n cohort cases. Wherever they apply, wages will grow with seniority even with multi-period employment. Needless perhaps to add, this conclusion rests on the simplifying assumptions embodied in the quit functions of eq. (1) and their extension to n periods. The effect of positive rates of interest on changing optimal wage scales will be stronger the longer the prospective employment.

Another crucial assumption of eq. (1) was that the possibility of second period quitting does not influence q_1. With expected future quitting, the utility at the beginning of employment is

$$(21)\quad U_o = U(w_1) + U(w_2)(1-q_2)$$

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The term $(1-q_2)$ in eq. (21) has a similar effect as the introduction of the discount factor α in eq. (13): seniority premium is not always optimal. The analysis is more complex since, unlike α, $(1-q_2)$ cannot be treated as a constant in the minimization procedure.

It is often argued and found in practice that wages in the training period are lower than alternative earnings and higher subsequently. With entry-point training, as in the present model, this need not be so. Firms with comparatively higher training cost may pay wages higher than others from the first day of employment, as Stiglitz (1974) demonstrated in his two sector analysis.

Conclusion

We have seen that with recruitment and training costs wages will be set to optimize turnover. The possibility of turnover will be reflected not only in the wage rate but also in the wage scale--giving rise to seniority premium. However, with positive interest rates, the optimal theoretic solution may well be a negative seniority premium.

Seniority payments are widespread and exist under varying economic circumstances. While we found that in the model presented here, rising wage scales are not always adopted. It seems, therefore, that cost of recruitment and entry point training can participate in determining wage scales but they are unlikely to be the sole factors in the prevalence of seniority premiums.

Firm specific training and the associated premiums create wage differentials which raise questions of general elquilibrium and welfare analyses. Another set of issues raised by the discussion in the paper is of the probability of layoffs and firing and their effects on quit rates and turnover. These questions are beyond the comparatively narrow scope of the present analysis.

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Appendix: Lagrangian Constrained Minimization

We show in this Appendix that optimality of seniority premium can be established using Lagrange multipliers.

A. Two Periods

The seniority premium is x, and define 2s = x; the wage rates are $w_1 = w - s$; $w_2 = w + s$. Write the Lagrangian

$$\text{(A.1)} \qquad H = R[T+(w-s)(1-q_1) + (w+s)(1-q_1)(1-q_2)] - \lambda s$$
$$= z - \lambda s$$

The control variables are w and s, and if $\lambda<0$ then $s>0$ and seniority pay is optimal at the unconstrained minimum. Differentiating

$$\text{(A.2)} \qquad \frac{\partial H}{\partial w} = \frac{\partial z}{\partial q_1}\frac{\partial q_1}{\partial w} + \frac{\partial z}{\partial q_2}\frac{\partial q_2}{\partial w} + \frac{\partial z}{\partial w} = 0$$

$$\frac{\partial H}{\partial s} = \frac{\partial z}{\partial q_1}\frac{\partial q_1}{\partial s} + \frac{\partial z}{\partial q_2}\frac{\partial q_2}{\partial s} + \frac{\partial z}{\partial s} - \lambda = 0$$

$$\frac{\partial H}{\partial \lambda} = -s = 0$$

At $s = 0$

$$\text{(A.3)} \qquad \frac{\partial q_1}{\partial s} = q_1'()[-U' + U'] = 0$$

$$\frac{\partial q_2}{\partial s} = \frac{\partial q_2}{\partial w}$$

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Also, note

(A.4) $$\frac{\partial z}{\partial s} = -(1-q_1)q_2 \quad <0$$

$$\frac{\partial z}{\partial w} = (1-q_1)(2-q_2) \quad >0$$

Rearrange (A.2), using (A.3)

(A.5) $$\lambda = \underset{(-)}{\frac{\partial z}{\partial s}} - \underset{(+)}{\frac{\partial z}{\partial w}} - \underset{(+)}{\frac{\partial z}{\partial q_1}}\,\underset{(-)}{\frac{\partial q_1}{\partial w}}$$

By the first order conditions, since the first 2 expressions on the right-hand-side in the first line of (A.2) are negative,

$$\frac{\partial z}{\partial w} > \left| \frac{\partial z}{\partial q_1}\frac{\partial q_1}{\partial w} \right|$$

And, in (A.5)

$$\left| \frac{\partial z}{\partial s} - \frac{\partial z}{\partial w} \right| > \frac{\partial z}{\partial w}$$

This proves that $\lambda <0$.

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B. Three Periods--Outline of Analysis

Define the wage rates as

$$w_1 = w - s_1;\ w_2 = w + s_1 - s_2;\ w_3 = w + s_2.$$

For seniority scale $w_1 < w_2 < w_3$, which imply

(a) $s_1, s_2 > 0$

(b) $s_2 > s_1/2$

(c) $s_2 < 2\ s_1$

The analysis is conducted in three stages. In the first stage, the constraint is $s_1 = s_2$. In the next stages the constraints corresponding to conditions (b), (c) above are imposed. Note that if $s_1 = s_2$, then $w_1 = w_2 = w_3$, and if $s_2 = s_1/2$ then $w_2 = w_3$, $s_2 = 2s_1$ implies $w_1 = w_2$. These equalities are used in the calculations of $\partial q_i/\partial s_j$ under the alternative constraints.

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FOOTNOTES

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1. The formulation of equations (1) to (4) is due to Collier and Knight (1986).

2. Except in cases in which the value of the marginal product was reduced unexpectedly and, in the judgment of the firm, temporarily.

3. For details see Collier and Knight (1986).

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